

**Response to ComReg's Line Share  
Consultation**

**Report**

**Non-Confidential Version**

**Prepared for**

**Eircom**

**By**

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## **Executive Summary**

This report sets out Indecon International Economics Consultants' and London Economics' assessment of ComReg's consultation paper on Line Share (LS) price regulation (ComReg 08/106). Much of the consultation paper is based on a consulting report for ComReg (ComReg 08/106a). Indecon does not agree with the conclusions of the consultation paper and, in some cases, the conclusions and the evidence of the consulting report (ComReg 08/106a).

In summary, ComReg is proposing that the share of fixed and common costs of the local loop in the rental price of line sharing in Ireland should be set to zero. (In other words, all the fixed and common costs would be recovered by voice line rental<sup>1</sup>).

ComReg's main arguments are based around the following:

1. The pure incremental cost approach is the 'best' approach, according to their consultants;
  - a. The incremental cost, given a subscriber has a voice (PSTN) line, is zero.
2. According to ComReg 08/106, the "most important and fundamental<sup>2</sup>" consideration is that eircom already is recovering its access network costs from voice (low frequency) line rental charges
  - a. Therefore eircom has no need to recover access network costs from LS rental charges.
3. Allocating zero of the fixed cost recovery to LS is not likely to negatively impact on the market, discriminate against certain technologies, discourage investment, competition, and technological development.

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<sup>1</sup> Our understanding is that ComReg is allowing some small incremental costs, such that the actual rental for line share would be €0.75.

<sup>2</sup> Page 8. "The most important and fundamental consideration is that the cost of a local loop on a bottom up long term incremental cost ("BU-LRIC") basis is already fully recovered through the price charged for narrowband access services ..."

In summary, we believe that ComReg is either incorrect or has not sufficiently proven its case to reverse its previous determination that the structure of line share/PSTN rental prices should be set on a 50/50 basis. Our position is based on the following reasons:

1. The incremental cost approach has not been shown to be the best; a variant of Ramsey pricing is likely to be the best in terms of both consumer and producer total value (welfare).
  - a. ComReg's own consultants have recognised that Ramsey pricing, or pricing that considers demand elasticities while neither under- nor over-recovering costs is the first best solution; however, they reject this as too difficult and not elsewhere implemented.
  - b. A vast body of international research suggests Ramsey pricing, or variants thereof, is the best, and that this is a very common form of pricing in the international network utility setting in OECD countries.
  - c. We estimate elasticities of voice and broadband services in Ireland. The estimates suggest that while uncertainty exists of the 'exact value', there is little uncertainty that voice services are sufficiently elastic such that allocating all of fixed access costs to them would be inefficient.
  - d. ComReg's claimed basis of their judgment to choose incremental costing (IC) as the pricing basis for LS is not sound. The claimed split of ComReg's own consultants is that 10/18<sup>3</sup> selected countries use IC. The methodology is highly arbitrary and sensitive to the sample. Inclusion of more natural comparators<sup>4</sup> to Ireland would mean only 10/22 countries use IC, and only 3/22 use a zero allocation of fixed access costs to LS.

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<sup>3</sup> The EU 15 plus the USA, Japan, and Australia.

<sup>4</sup> Including NZ, Canada, Switzerland, and Norway.

2. ComReg's current claim that tariff rebalancing implies a zero cost share for LS in the structure of charges is logically unfounded; the efficient *level* of cost recovery tells one nothing in particular about the efficient *structure* of costs.
  - a. The level of costs is not a relevant issue for this consultation *per se*; the structure of costs is. Eircom's overall level of cost recovery for its access network and the *most efficient structure of charges* between PSTN and LS are two wholly separate issues.
  - b. ComReg should, according to its duties and responsibilities, determine independently:
    - i. The efficient levels of cost recovery for services which are joint and common to fixed network assets
    - ii. The efficient structure of charges—which should include some element of demand-side analysis when determining the efficient share of common cost recovery.
  - c. Indecon/LE submit that correctly, Eircom's sole concern here is with the structure of charges/the split of fixed cost recovery between voice and LS. Eircom merely have a legitimate concern that voice line rental is rapidly becoming a more elastically demanded product and they need pricing flexibility to react to market dynamics and uncertainty<sup>5</sup>. [REDACTED]  
[REDACTED]  
[REDACTED]  
[REDACTED]

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<sup>5</sup> As evidence of this, eircom note to us the fact that they did not raise PSTN prices by the 5% CPI increase allowed in October 2008 under the current price cap. We suggest that this is clearly a 'revealed' evidence of eircom's view of the increasing price elasticity of voice line rental.

3. ComReg's current proposal that LS rental charge access network cost share of zero is contrary to ComReg's own objectives, as it is likely to negatively impact the market, discriminate against certain technologies, and risks 'choosing winners' (e.g., LS over Cable BB, Mobile BB, Satellite BB, FTTH, etc), negatively impacting particular operators and technology users, and is overall likely to be inefficient<sup>6</sup>,  
<sup>7</sup>.
  - a. The evidence produced by ComReg's own consultants ComReg 08/106a suggests low LS prices reduce investment.
  - b. A LS price that is artificially low will negatively impact Cable Broadband (BB), which is both the fastest growing part of the fixed BB market in Ireland and in many countries the most significant BB competitor.
  - c. Eircom proposes that a reasonable approximation to a Ramsey pricing policy could be reached within a reasonable time with ComReg. The approach would be based on agreement on a) the overall *level* of charges/cost recovery (akin to the Ramsey number) and b) the structure of charges (based on ratio of inverse elasticities).

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<sup>6</sup> One of ComReg's objectives under the Communications Regulation Act 2002 is to promote competition including ensuring that there is no distortion or restriction of competition.

<sup>7</sup> The inefficiency arising from the fact that demand elasticities have not been fully exploited in the setting of the share of fixed and common costs to be recovered between two joint products: voice and LS.

# 1 Introduction

## 1.1 Background

Line Share ('LS') is the provision of access to the local loop, in which the local loop will support telecoms services of other authorised operators ('OAO's) as well as the fixed-line provider (eircom).

LS prices have been set since 2001 in Ireland so as to support 50% of the local loop costs on a per unit basis (the absolute or total share of local loop costs on the PSTN (voice) access network is very high, as LS numbers are low and voice penetration is high—however, LS has been growing rapidly while PSTN customers have been falling). In 2007, ComReg found that there is no longer an access deficit in Ireland and tariffs were accepted as being fully rebalanced (between voice rental and call minutes<sup>8</sup>). Since 2007, ComReg tried to lower the LS price from €8.41, but this was reversed on appeal. Subsequently, ComReg concluded that applying such a 50:50 allocation rule of local loop costs between broadband and voice services without any discount on the PSTN (public switched network) rental charge could lead to a cost over-recovery by eircom, if LS prices continued to grow. Voice PSTN rental lines, however, are falling rapidly.

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<sup>8</sup> It is noteworthy that part of the economic rationale for rebalancing was that mobile call charges and PSTN call charges needed to be put on an equal basis for fair competition, as well as that the elasticity of rental was lower than the elasticity of call minutes. Similar considerations should apply here.



ComReg then commissioned consultants (TERA) to assess the possible methodologies for the allocation of local loop fixed and common costs in the context of the current situation.<sup>9</sup> TERA recommended implementation of an ‘incremental’ methodology leading ComReg to conclude a zero allocation of local loop fixed and common costs to the LS monthly rental price, claiming that this methodology is adopted by most of the National Regulatory Authorities (NRAs) in the countries reviewed in the report. TERA further advised, based on a “*preliminary assessment*” of the relevant incremental costs related to LS in Ireland, setting the monthly cost-oriented price for LS at a maximum price of €0.75<sup>10</sup> per line per month (Indecon/LE’s emphasis).

At a high level, ComReg’s/TERA’s, logic may be summarised as follows.

1. The pure incremental cost approach is the ‘best’ approach, according to their consultants;
  - a. The incremental cost of the fixed line access network, given a subscriber has a voice (PSTN) line, is zero (small variable costs are allowed giving €0.75).
2. According to ComReg 08/106, the “most important and fundamental<sup>11</sup>” consideration is that Eircom already is recovering its access network costs from voice (low frequency) line rental charges
  - a. Therefore eircom has no need to recover access network costs from LS rental charges.
3. Allocating zero of the fixed cost recovery to LS is not likely to negatively impact on the market, discriminate against certain technologies, discourage investment, competition, and technological development, etc.

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<sup>9</sup> ComReg08106a.pdf (23 December 2008).

<sup>10</sup> Our understanding is that there are some incremental costs to line share such as the need to fix additional faults on the line, etc, and this is the source of the 0.75. According to eircom, “carrier removal is the only item allowed. The are additional itema around product development and faults, and billing and administration but we do not comment on these. There is some efficient level of incremental cost related to these items but we have not analysed this in detail.

<sup>11</sup> Page 8. “The most important and fundamental consideration is that the cost of a local loop on a bottom up long term incremental cost (“BU-LRIC”) basis is already fully recovered through the price charged for narrowband access services ...”

## 1.2 Purpose and structure of the response

The purpose of this report is to respond to ComReg's consultation on LS pricing.

Indecon's response document is organised as follows:

1. Section 2 Responds to ComReg's main arguments
  - ComReg has not provided sufficient evidence that the incremental approach is better than the Ramsey approach, advocated by its own consultants.
  - Responding to the claim that the 'level of cost recovery' implies a zero cost allocation for LS – the level of cost recovery and the efficient structure of charges are two separate issues.
  - Responding to other arguments and the flawed conclusions of ComReg regarding tariff rebalancing and the impact of too low a cost on investment- pricing line share inefficiently low will not promote BB in Ireland, and will be contrary to ComReg's own principles and duties.
  - A point by point discussion of ComReg 08/106 is undertaken
2. Section 3 gives a detailed discussion and investigation of Ramsey pricing and other cost sharing/allocation methodologies.
3. Section 4 gives empirical evidence on price elasticities for local loop services in Ireland (Voice, broadband, business residential, etc).
4. Section 5 gives Conclusions
5. Additional information is in the Technical annexes

## **2 ComReg's current positions and Indecon/LE's main arguments**

In summary, we believe that ComReg is either incorrect or has not sufficiently proven its case to reverse its previous determination that the structure of line share/PSTN rental prices should be set on a 50/50 basis. This section sets out our main arguments and responses.

### **2.1 The pure incremental cost approach has not been shown to be better than variants of Ramsey**

#### **2.1.1 Ramsey pricing is best and can be approximated in the current context**

Ramsey pricing is held to be the first best option (if practical)—we concur with ComReg and ComReg's consultants, and other sources on this conclusion. However, the only real problem according to TERA is it is difficult to implement. Seemingly, if the implementation problems went away, then, this would be the recommendation of TERA.

We discuss Ramsey pricing in detail elsewhere in the report. However, at a high level, the logic Ramsey pricing can be understood as the following. Ramsey pricing says, "If the regulator/policy maker is to recover fixed or joint and common costs, and minimise consumer welfare loss from the need to price above marginal cost—how should it be done? The answer is to set price-cost margins or shares of fixed and common costs to be recovered in proportion to the inverse of demand elasticities." In other words, products which are more price sensitive should bear a lower burden of the cost recovery, in order to minimise the reduction in consumer demand associated with the higher price.

Ramsey pricing has been widely used in the utilities context. Peak-load pricing, is a special example of Ramsey pricing, for example, for which the concept is sometimes called Ramsey-Boiteux<sup>12</sup> pricing.

TERA seemingly did not fully consider the possibility of the global price cap, as a means of implementing Ramsey prices, and as recommended by Laffont and Tirole (2000)<sup>13</sup>.

The global price cap rule basically considers the retail price(s) and access charge(s) in one single price cap formula. Once the price cap is determined, the incumbent is free to set its access charge(s) and retail price(s), so long as the global price cap is satisfied. The idea is essentially equivalent to the regulator imposing a price cap on a basket of goods/services provided by the incumbent, including the access service(s). Since the incumbent is free to determine the price(s) of access and the prices of its other goods/services, the global price cap can be viewed as a form of 'decentralised' Ramsey pricing. The main perceived benefits of price caps generally are: simplicity; they incentivise incumbent operators to enhance efficiency; and they provide regulators with updated information about efficiency potential over time.

Laffont and Tirole (2000 p132) state:

"The *structure* of unregulated firms' prices (though not the level if the firms have substantial market power) thus reflects Ramsey-Boiteux precepts. This observation suggests that the most promising alley for implementing Ramsey pricing in a regulatory context is to decentralize pricing decisions to the operator.

The idea of decentralizing pricing decisions may be foreign to those who favour heavy regulatory intervention. Yet, a key feature of the regulatory revolution of the 1980s was departure from the detailed setting of individual prices and flexibility to operators to adjust their price *structure* to demand and competitive pressure conditions. While the implications of this revolution for access prices have been overlooked..., we still find it surprising that regulators who routinely design price caps dismiss-offhand Ramsey pricing as being informationally infeasible!"

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<sup>12</sup> Boiteux was a prime developer of the concept in practice for EdF in the 50s. EdF, when switching to a majority of nuclear capacity, needed an efficient pricing scheme to recover the very high proportion of fixed costs (but low marginal cost) associated with a large nuclear production plant base – peak load pricing was the answer. It is noteworthy that this also was efficient in terms of consumer benefits.

<sup>13</sup> Jean-Jacques Laffont and Jean Tirole, *Competition in Telecommunications*, MIT Press 2000, page 174.

The availability and knowledge of this method is significant, and it is recommended as one option to the retail and wholesale split of access charges, according to the ICT<sup>14</sup>.

“Wholesale and retail prices are interlinked. Regulators need to be aware this link in regulating prices.

One option to address the link between wholesale and retail prices is to implement a double price cap (also known as a “global price cap”).

Under a global price cap plan, the regulator includes wholesale interconnection services in the price cap plan, and treats them as any other final good in implementing the price cap.

For example, in the United States, some state regulators control intrastate long distance interconnection prices through the same price cap mechanism that regulates retail prices. These price cap plans are partial examples of a global price cap; they are not full global price caps because local interconnection prices are regulated using cost-based price controls.

According to Laffonte and Tirole, a global price cap plan can incorporate a Ramsey pricing structure and has the following features:

The wholesale service (access) is treated as a retail service and is included in the computation of the price cap, and

Weights used in computing the price cap are determined exogenously and are proportional to forecast quantities of the associated services.”

As Laffonte and Tirole state, “global price caps thus enable regulation to be more light-handed, for global price caps reduce perverse incentives and therefore diminish the need for regulatory oversight of the operator’s decisions.”

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<sup>14</sup> See the ICT Regulatory toolkit online. <http://www.ictregulationtoolkit.org/en/Section.2163.html#End1>

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It is also noteworthy that ComReg in some of its recent history has explicitly considered demand elasticities and how fixed and common costs should be allocated based on demand-side as well as cost-side considerations. For example, in ComReg 05/94, ComReg explicitly argued that demand elasticity impacts from proposed rises in stamp prices could impact on postal services and the ability to fund the USO negatively. More specifically, ComReg was concerned that shifting too much of the joint and common cost burden of the mail service to stamped letter mail from bulk mail would lead to significant volume reductions. Indecon presented estimates that letter mail was in fact less elastic than bulk mail. ComReg stated, "An Post's revised elasticity report, which cannot be published by ComReg because An Post has argued it contains commercially sensitive information, suggests that ComReg's initial concerns about the impact of price increases on volume were not without foundation." ComReg, while questioning somewhat the statistical method, was concurring that the elasticity approach was an important consideration in deciding the split of prices/costs between bulk and stamped services. This is very analogous to the current consultation.

In general, Ramsey pricing, although not necessarily implemented to the letter, is almost always considered in that the general notion of demand elasticities is considered when setting prices. This is true in setting all kinds of utility prices (for example, in Ireland, rail prices have peak and off-peak, a commuter belt ticket is quite a bit less expensive than an intercity ticket, postal prices are set with respect to demand elasticities, etc).

Finally, we note that the TERA report claims that their estimates and conclusions are 'dynamic and consistent with the Ramsey-Boiteux principle.

methodologies and is consistent with the Ramsey-Boiteux methodology (see Section 5.1).

***From both a static and a dynamic point of view, not allocating any common costs of the local loop to LS is the preferred methodology (the "incremental" methodology as opposed to the "50:50" methodology).***

It is not clear to us *how* they have determined that the conclusion 'not allocating any cost of the local loop to LS' is 'consistent with the Ramsey-Boiteux methodology'. They have not undertaken either a review of the literature on elasticities or estimated the relevant elasticities themselves. Their document does not discuss the detail of how their preferred proposal takes into account Ramsey principles or elasticities.

Further, their criterion for 'dynamic' appears at odds with industry practice and economic standard. Apparently, 'dynamic' means that 4/18 regulators have *switched* to this methodology. We would submit that this is highly arbitrary, and would be stronger evidence that 14/18 *did not* switch to this methodology<sup>15</sup>. More generally, we would suggest that the proper basis for the needed dynamic considerations, would be to study the dynamics of the elasticities and demand conditions in the relevant markets going forward.

Further, in reviewing their own list of pricing options, ComReg/TERA have apparently elevated 'simplicity' of implementation to the highest importance in regulatory pricing decisions. There is no apparent detailed discussion, although it seems that implicitly, this is their own basis for elevating their current proposal over the Ramsey-Boiteux solution. While we accept that if any form of Ramsey-Boiteux pricing were completely 'infeasible', then their conclusion would be correct, they have not made the explicit argument that Ramsey-Boiteux pricing principles would be impossible to apply. We argue that they are not impossible, nor exceedingly difficult to apply in any case. Further, we argue that telecoms pricing in the general sense is a complicated field, with large financial stakes, and detailed experts involved, and that therefore simplicity should rank relatively *low* on the decision analysis scale of ComReg.

While studying in detail the dynamics of elasticities involved was beyond our scope, qualitatively, judgment would suggest that the market for telecoms service and relationships between price and demand for broadband, voice, and related products is very dynamic (e.g., changing over time). A large driver of the relative elasticities will be changing consumer tastes as well as the technology that enables one service to be more substitutable for the other. It seems apparent, based on declining PSTN numbers (both overall, e.g., retail plus wholesale, and retail alone) indicates that demand might be becoming more price elastic.

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<sup>15</sup> In addition, experience with some of these cases has been seemingly interpreted by ComReg. According to industry sources, Belgium never really had 50/50 to switch from. Demanrk explicitly considered moving from 50:50 to incremental, but decided it was wrong. Whether these points are valid may be checked with deeper consideration, but our point is that ComReg is seemingly interpreting events.

We conclude this sub-section that some form of 'consideration of the demand elasticities' is required in setting the efficient level of costs to be shared by joint and common services. The proposals of ComReg are not consistent with the Ramsey-Boiteux principles. Demand-side conditions can, have been in the past, and should be in the future, considered in detail when allocating fixed, joint and common costs. We do not agree that the 'dynamics' of the cases studied are supportive of ComReg's position. A detailed study, or at least qualitative comments, on the dynamics of the PSTN and BB markets in Ireland, and their relative demand elasticities is what is required.

## 2.2 The efficient level of prices does not imply the efficient structure of prices

ComReg, in our opinion, misapplies two key elements of price regulation: the efficient *level* of prices, and the efficient *structure* of charges. As evidence of this, they cite at the beginning of the section 5 in ComReg 06/108 the Arcor case, citing that cost orientation "prohibits...remuneration several times for providing the same service".

We would submit that a) this has nothing to do with cost structure and cost allocation, but the *level* of cost recovery; b) eircom is not seeking to recover costs "several times over"; and c) eircom is not providing the 'same service' but two different services. The cited case is clearly talking about the *level* of cost recovery for a *single* service, whereas here, the main issue is the structure of cost recovery between *two* services.

The very concepts of prices, price levels, and price structures, go to the foundations of price theory and mathematics. Any set of prices can be seen to be a vector in N-space. The vector has both a magnitude and direction. The level of the prices is akin to the magnitude and the structure akin to the direction. In this case, the vector is a two dimensional vector, giving the prices of LS and PSTN rental; the magnitude can be separate from the direction. Suppose that the current 50/50 rule is implemented as the correct structure of prices (the direction – in 2-space, this is the 45-degree line): then it can be shown that an infinite set of prices,  $p_1$ ,  $p_2$ , is possible (all points on the 45-degree line), but only one that lets total cost equal total revenue exists:

Solving for the efficient price level ( $R=C$ ) or the price structure (if allocation rules are known) is not difficult. Suppose the following,  $R$  is revenue, and  $C$  is total cost:



**Equation 1**

$$R = p_1 q_1 + p_2 q_1$$

**Equation 2**

$$C = \frac{C}{R} * R = \frac{C(p_1 q_1 + p_2 q_1)}{R} = C \left( \frac{p_1 q_1}{R} + \frac{p_2 q_1}{R} \right)$$

The equation says the revenue share weighted average of cost will give the appropriate pricing *level* to allow costs to be recovered.

The very general Ramsey pricing methodology makes the distinction between cost recovery and cost structure via the "Ramsey number". Ramsey pricing in the simple case is:

**Equation 3**

$$\frac{p_i - c_i}{p_i} = \frac{\rho}{\epsilon_i}$$

Here, p is price, c is marginal cost, epsilon is the elasticity, and rho ( $\rho$ ) is the Ramsey number. This number is adjusted up or down to adjust for the level of fixed costs to be recovered. If the level is high, the Ramsey number is adjusted upwards and vice versa, so that only cost is recovered.

We submit that little discussion of the structure of charges is given by ComReg which is dichotomised from the level of charges, and such consideration should be required to set the structure of charges.

The efficient structure of charges is given by a Ramsey-type pricing formula.

This means that consideration of the demand elasticities of various products should be considered. According to research by Millar (2007) focus can be limited to the own price elasticities. According to Indecon's own research (see the detailed section on price elasticity estimation, own price elasticities of voice and broadband tend to be similar. Our preferred model suggested own price elasticity of -0.626 for DSL service and -0.696 for PSTN (both retail). This is suggestive of the 50/50 rule, although our main point is to advocate that the (0,1) allocation is incorrect, rather than the 50/50 rule is correct.

However, eircom, and judgementally Indecon, believe that dynamically and going forward, DSL, and PSTN for fixed line networks are likely to become more elastic, and that DSL might still be somewhat more elastic than PSTN; this could change, however. There are also possible dynamics with complementarity between a variety of products going forward that might be important. We have not fully investigated this as this would require modelling interrelated demands of voice, TV, mobile, and DSL. Qualitatively, though, we argue that this investigation is not necessary, or alternatively, could be done with additional time and effort.

Further we submit that ease of implementation of the overall level of charges is no different from the (1,0) cost allocation proposal of ComReg versus our alternative proposal. If, as ComReg claim, there is greater risk of over-recovery than under-recovery, then ComReg should be checking the overall level of prices for PSTN, LLU, and LS together on a regular basis<sup>16</sup>. The market is sufficiently dynamic that this cannot be ignored. This is supported by the TERA report—stating rental plus LS should not over-recover. The probability that either over or under recovery occurs could change quickly due to demand and supply conditions, changing technology, etc. This also points to the need for some flexibility and a light-handed regulatory approach.

It is also important to note that our arguments are based around cost recovery for the PSTN network as a whole. Any arguments that the efficient structure of charges should be considered on a loop-by-loop basis would, in our opinion, be nonsensical for the following reasons. First, one would have to consider detail about elasticity and demand on a loop-by-loop basis. Further, as eircom has a USO, and has noneconomic customers and exchanges, it is clear that policy dictates that individual lines are not set so as to recover costs. The very existence of a sustainable USO points to the conclusion that a loop-by-loop basis for cost recovery is not relevant.

In conclusion to this subsection, perhaps we should state that we agree fully with ComReg's own consultants conclusions on this point, and suggest that ComReg should consider taking the advice of TERA in ComReg 08/106a. TERA concludes:

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<sup>16</sup> Alternatively, mechanisms for 'true-ups' over time exist in other regulated markets for telecoms and in Ireland (e.g., CER has correction factors to true-up over or under-recovery of gas charges for BGES).

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*When tariffs are rebalanced between access and fixed line calls, if some access network costs are allocated to LS according to the "50:50" methodology, these costs should be deducted from the retail PSTN monthly rental charge.*

More generally, we would conclude that ComReg, consistent with its own consultant's report, should set the efficient level of charges *and* the efficient structure of charges ("these costs 'should be deducted' from the retail PSTN..charge"). If the dynamics and changes in the elasticities of voice and LS indicates that the level of PSTN charges should come down if voice is falling and LS is rising, then that is what should be studied and, if needed, price adjusted accordingly to set the efficient 'structure' of charges.

A practical and implementable similar rule to Ramsey pricing can be found. In fact, as our empirical evidence shows, the Ramsey pricing rule might be akin to the 50/50 rule, while dynamic impacts might suggest some flexibility around the 50/50 rule, and perhaps the global price cap approach. We suggest that a negotiated process and perhaps a complete investigation of the elasticities is warranted, but that sufficient evidence and correct logic for the current proposal has not been found by ComReg.

### 2.2.1 Discussion of incremental cost arguments methodology

While there may be a risk of over-recovery in the 50/50 rule, our opinion is that ComReg has not fully studied the dynamics of the market and the nature of over or under-recovery as would be impacted by their decision. There is always a risk of over recovery, and also a risk of under-recovery. With ComReg's current proposal of no recovery of common costs for LS, the risk may be tilted towards under-recovery in the future, if revenue from voice line rental falls at all significantly (whereas with some allocation to LS, there would still be a risk of under-recovery if PSTN revenue fell faster than the increase in LS revenue).

While it is true, that there may be a risk of over-recovery with the current cost levels *and* structure, there are dynamic considerations that seemingly have not been covered.

First, it is important to consider that there is a more general real risk of under-recovery with the incremental costing methodology.

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Incremental costs usually involve economies of scope and other pricing challenges in the general LLU or access pricing problem. While ComReg accepts that some incremental costs exist (including Carrier removal, product development, fault repair, and product development), ComReg then adjusts eircom's estimates of these, or claims the cost is recovered elsewhere or was already covered in the past. These costs should be added to an allocation of common costs of the loop<sup>17</sup>.

Economies of scope, which refer to cost savings arising from the production of two or more products/services at the same time, are relevant to access pricing problems in telecoms generally owing to what they imply for incremental cost. Therefore, it is useful to outline the concept (and that of incremental cost) in more detail. To do that, we first need to define some fundamental cost concepts.

Assume that a telecoms operator provides three products or services,  $X$ ,  $Y$  and  $Z$ . The stand-alone cost of providing a given service is the total cost of producing just that service, when the output of all other services is zero. Thus, the stand-alone cost of providing  $X$ , denoted by  $C(X)$ , is given by:

**Equation 4**

$$C(X) = TC(X, 0, 0)$$


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Similarly, the stand-alone cost of, say,  $Z$  is given by  $C(Z) = TC(0, 0, Z)$ . In each expression,  $TC$  denotes total cost.

The marginal cost of  $X$ ,  $MC(X)$ , is defined as the extra to total cost resulting from increasing output of  $X$  by one more unit. Mathematically,  $MC(X)$  is given as the partial derivative of the total cost of producing  $X$ ,  $Y$  and  $Z$ ,  $TC(X, Y, Z)$ , with respect to  $X$ , viz.:

**Equation 5**

$$MC(X) = \frac{\partial TC(X, Y, Z)}{\partial X}$$


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<sup>17</sup> It could be argued that a similar methodology to what we are advocating is followed for full LLU--the LLU price includes LRAIC plus mark-up.

Incremental cost is not necessarily the same as marginal cost. Incremental cost is defined as the addition to total cost resulting from the entire supply of X. Formally, the incremental cost of X,  $IC(X)$ , is defined as the difference between total cost with and without X supplied, namely:

**Equation 6**

$$IC(X) = TC(X, Y, Z) - C(0, Y, Z)$$

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Note that for a one-unit supply of a product or service, incremental cost coincides with marginal.

Economies of scope occur when the cost of providing a given service (X) along with other services (Y and Z) is less than the cost of providing X on its own. In terms of the above, economies of scope arise when the incremental cost of providing X is less than the stand-alone cost of providing X, namely:

**Equation 7**

$$IC(X) < C(X)$$

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Starting from the point where it provides all three products, a telecoms regulator seeking to establish the incremental cost of X essentially needs to establish what costs the operator could avoid were it to cease provision of X. If it ceased to provide X, it would accordingly reduce its variable costs in the short-run. Thus, in the short-run, incremental cost and variable cost are in general equivalent. In the long run, however, since the fixed costs required for X (but not for Y and Z) can also be avoided, the incremental cost of providing X will be higher than the variable cost associated with X. This implies that the long run incremental cost or LRIC of supplying X will be greater than variable cost.

In the context of LS, we have no difficulty with the theory that given the voice line is rented, then the LS incremental cost is zero. The difficulty arises in practice since there is no way to determine 'which service comes first'.

### ***Long Run Incremental Cost (LRIC) and over or under recovery***

ComReg seemingly give excessive weight to the argument of the risk of over-recovery, but devote little space to the risks of under-recovery. There are a number of possible means by which under-recovery might occur.

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It is not clear that charging the incremental cost could not lead eircom to *under-recovery of total costs*, especially if the number of voice subscribers continues to fall as it is doing now. Specifically, eircom's risks of under recovery include: 1. the network business is cost invariant to the number of loops in service; 2. costs per loop are set to recover Network cost divided by loops in service at any given point in time; and 3. even if the correct price per loop is set at present, any inefficiencies in the pricing structure that reduce future demand will result in under recovery.

The LRIC of a service is the increase in a firm's total cost that arises when that service is added to the firm's production mix and the firm can adjust the fixed costs associated with that service. Put another way, the LRIC is given as the firm's total current cost less the cost it would incur if it were to end the service whose incremental is being considered. Following our earlier mathematical treatment if the firm produces service  $x_1$  and adds  $x_2$  to its output, the LRIC associated with  $x_2$  is given as:

**Equation 8**

$$LRIC(x_2) = C(x_1, x_2) - C(x_1, 0)$$

where  $C(x_1, x_2)$  is the total current cost from providing both services and  $C(x_1, 0)$  is the stand-alone cost of providing  $x_1$ . LRIC has the virtue that the cost is entirely attributable to the service in question. However, the difficulty with LRIC, which also besets FDHC, is the presence of common fixed costs that are not attributable to any one service and consequently do not enter the LRIC of that particular service. Access prices set equal to LRIC (or close to LRIC) in effect deal with common fixed costs by ignoring them – they will be lost in the LRIC modelling. The issue is not a question of the difference between FDHC and LRIC (i.e., choice between historical or long run costs, forward looking costs, etc, does not remove the problem.) Clearly this will be inefficient for a firm that has substantial common fixed costs.

To demonstrate this potential pitfall of LRIC, consider the sum of the LRICs of the two products above, *viz.*:

**Equation 9**

$$\begin{aligned} LRIC(x_1) + LRIC(x_2) &= C(x_1, x_2) + [C(x_1, x_2) - C(x_1, 0) - C(0, x_2)] \\ &\leq C(x_1, x_2) \end{aligned}$$

The inequality obtains because the term in square brackets will generally be negative due to the presence of economies of scope (i.e., the joint cost of producing the two services,  $C(x_1, x_2)$ , will be less than the sum of their stand-alone costs,  $C(x_1, 0)$  and  $C(0, x_2)$ ). This means that some proportion of common fixed costs among the two services will not be recouped or accounted for by LRIC modelling, meaning that pricing on the basis of LRIC *alone* could be financially ruinous for such a firm (i.e., a firm with large common fixed costs).

It is noteworthy that full LLU is based on the LRIC concept (as well as other products). If all the prices are based on LRIC, there is a greater risk of under-recovery. ComReg does not explicitly consider the case above. They merely assert that currently, there is no access network deficit. However, our conclusion is that this needs to be studied in the context of the dynamics of the market—rising LS numbers and falling PSTN lines, and the application of the LRIC concept to individual products.

The result is that cost theory tells us that setting the overall level of prices has to be done on a joint production basis, and not on a product-by-product LRIC basis.

## **2.3 The efficient level of common cost allocated to LS rental prices is not likely to be zero (and the efficient level of costs allocated to voice is not likely to be 100%)**

### **2.3.1 Significant estimates of BB elasticity imply non-zero cost share for LS**

While it is difficult to get full precision on the elasticities involved, it is not difficult to be confident that the relevant elasticities are neither infinite nor zero (which would be what is needed to show a zero-one cost allocation plan of ComReg is optimal). It is also possible to estimate elasticities and study their statistical differences from zero (or infinity).

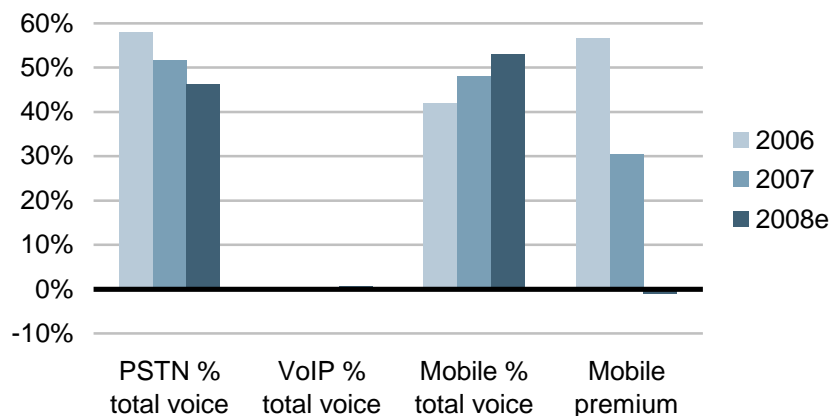
The efficient pricing-cost allocation decision is driven by the Ramsey/inverse elasticity rule. Indecon (and eircom) are in full agreement with ComReg and ComReg's consultants, TERA, and many prominent academics on this point. The point of difficulty in estimating elasticities made by TERA is noted, but we submit that we have sufficient evidence in our own elasticity estimates to provide a sound basis that the LS-price's share of costs should not be zero.

We estimated a number of models in our study of elasticities involved and all the models suggest own price elasticities of between -0.6 to -0.7, for both BB/DSL and PSTN services. Details are in the section following on the elasticity estimation.

### 2.3.2 Too low a LS price may penalise non LS-based competitive OAO strategies

It is evident from our own experience and results and international experience that infrastructure investment is coming from a variety of sources: Mobile BB, Cable BB, and full LLU.

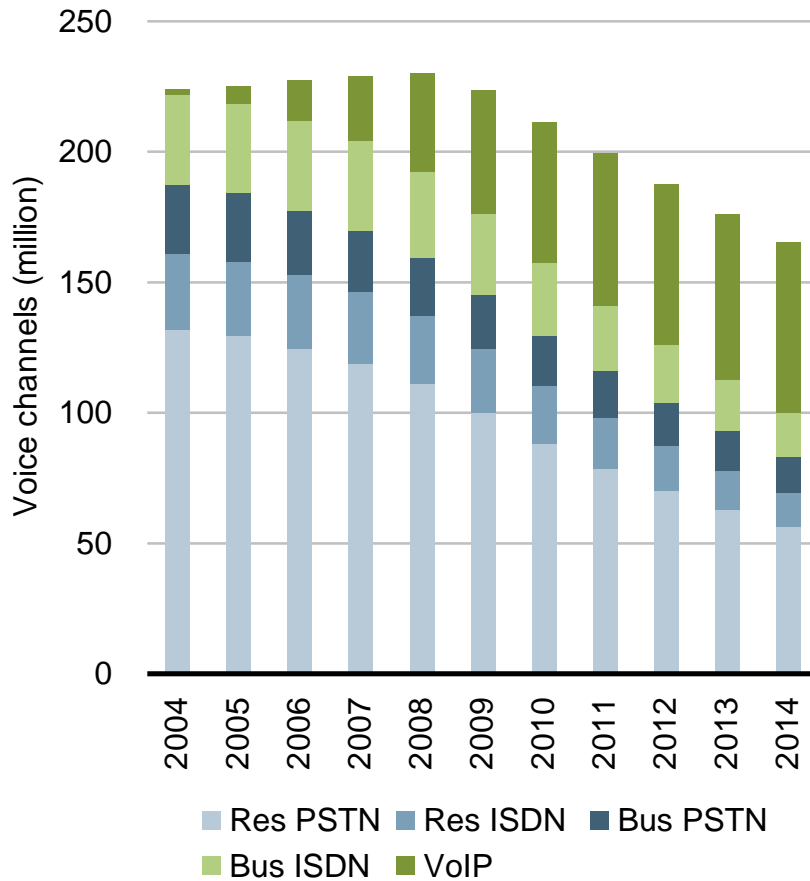
Overall, the evidence is that households are starting to drop voice for mobile only communications. According to reports provided to us by eircom to review (Analysys Mason), the following are the estimated trends in Ireland: PSTN now has a smaller and falling share of voice.



Source: *Analysys Mason*

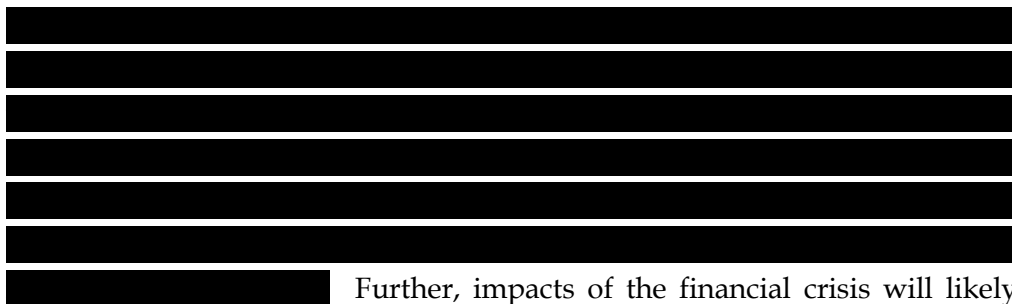


Overall, volumes of voice channels are expected to reduce in markets across the EU. Analysys Mason has forecasted the numbers of voice lines:

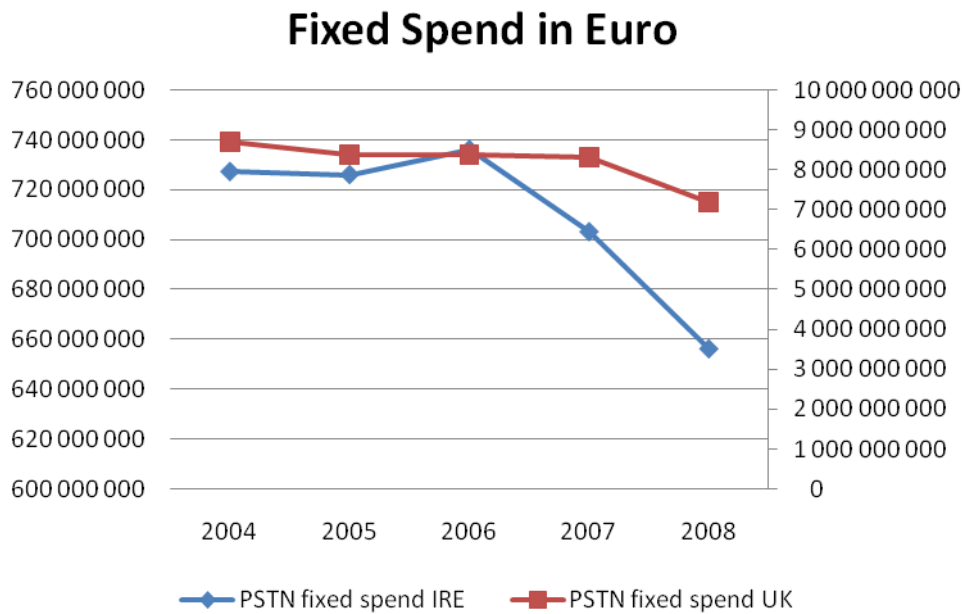


Source: Analysys Mason

Indecon/LE have also analysed historic trends using the Analysys data. The below graph shows that total fixed spend on PSTN is falling in both Ireland and the UK, but that the rate of reduction in Ireland is much more rapid.



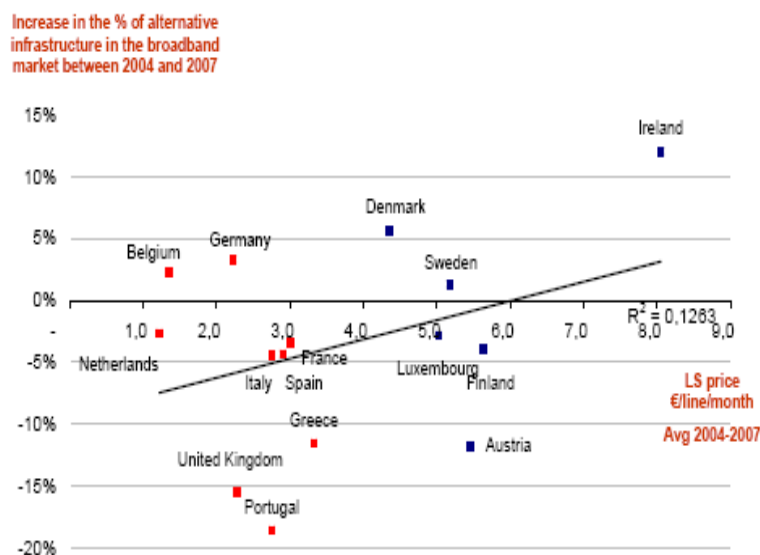
Further, impacts of the financial crisis will likely accelerate this, as indications are that budget conscious households are abandoning voice for mobile only telephony.



Source: Indecon elaboration of Analysys Mason/eircom data

ComReg discusses the point of whether infrastructure investment will not be affected by its current proposal. The study of the 'make or buy' decision is taken up in ComReg 08/106a.

The main part of the empirical analysis is apparently based on EC report data and is a basic single variable linear regression model (Figure 10). It is difficult to see how TERA has concluded that there is no impact on investment, as the slope of their estimated line is positive. The R-squared indicates that there *may* be statistical significance, but the full statistical properties of the model are apparently not explored, which is inconsistent with international best practice. We would submit that ComReg should interpret its own consultant's results in a transparent manner, so that third parties can ascertain the validity of the results. The figure below is replicated from ComReg 08/106a (figure 10). The seeming indication is that the increase in LS prices gives rise to an increase in infrastructure investment; while TERA asserts that the correlation is zero, the line is clearly positively sloped.



Source: TERA Consultants from progress report on the single European electronic communications market 2007 (13th REPORT) of the European Commission

Source: ComReg 08/106a

More details would also be helpful. For example, it is not clear why averaging the data is useful, since preserving the panel nature of the data would likely have provided more power and efficiency to the estimates. Casual observation suggests this might have been very interesting, because within and between group estimation (grouping by the LS methodology) could have been done. Casually, it appears that the blue group has higher investment growth on average. Thus, we submit that by the standards of the evidence presented (i.e., no statistical inference), Figure 10 suggests both the method and the price of line share as chosen by ComReg are more likely to negatively impact alternative infrastructure investment.

TERA studies the possibilities of alternative infrastructure investment on pages 29 and 30 of ComReg 08/106a. First, the situation discussed is fully consistent with our opinion that the level of cost of the network should not be over-recovered based on line share pricing (the line share price should be deducted from the monthly rental). Further, if in fact no over-recovery occurs, then distortion may not be an issue. However, there is no discussion of the efficient structure of charges; the full set of possibilities is not fully explored and the relative optimal level of the split of charges is not discussed.

The main case that is not explored by TERA is the case where alternative investments are needed to provide say, broadband services, but that significant portions of investment for cable, satellite or mobile have already been made. In this case, mobile or other potential alternative infrastructure providers might consider the pure marginal cost of BB versus the LS price in the make or build scenario. In other words, providers might provide cable TV (or satellite TV) and BB services together, but the price of competing BB service through line share could be crucial to the viability of these services. The relevant comparison is between the retail price of LS and the pure marginal cost to cable/satellite TV BB providers.

Considering the dynamics and uncertainty of the market over future technologies, etc, is also not considered in the TERA analysis. For example, suppose, via some unforeseen technological innovation, that Cable BB can provide a much higher speed/better quality/lower overall price service at some time in the future, but this is dependent on investments made today. In this case there could be 'path dependence' in the adoption of technology. Even though Ireland as a society would like to be able to switch to the new technology, because of fixed and/or sunk investments, it is not economic and/or not feasible. Examples of this are common in telecoms and communications where technological change is rapid, where early adopters fell behind or an inferior technology became the standard because of network effects. A celebrated example is the USA/Canada mobile phone technology versus the EU. Mobile penetration, quality and cost were all lower in general in the US than in the EU due to a different path, even though the USA was the early adopter of mobile technology.

### **2.3.3 Indecon's estimates of elasticity would imply greater consumer surplus than ComReg's**

In general, since the Ramsey pricing rule is the optimal rule for the allocation of costs when fixed costs must be recovered, deviations from the Ramsey rule would imply reductions in consumer welfare or consumer surplus. We would note that in fact, ComReg's obligation is to maximise welfare, that is, consumer plus producer surplus (value). Even if we restricted our welfare discussion to consumer surplus, it is unlikely that the relevant elasticities for PSTN and BB are zero and infinity.

In general, since the Ramsey rule is an optimal rule, and since consumer surplus is a smooth and well-behaved function of price and quantity (i.e., no jumps or discontinuities, increasing as price is falling, etc), then it can be generalised that deviations from the rule will be sub-optimal, and that being further from the 'correct' Ramsey prices will imply lower consumer surplus.

ComReg have not estimated the elasticities involved. However, intuition alone can inform us as to how likely they are to be correct. Consider the following. If ComReg's proposal were correct, then this would imply that the elasticity of PSTN was zero and/or that the elasticity of LS was very high (approaching infinite). However, as PSTN total lines have fallen with recent price rises, clearly, PSTN has some price sensitivity, so the elasticity is not likely to be zero. Conversely, BB lines have been growing as prices have come down, but not as rapidly as some might have hoped. The implication is that BB uptake is unlikely to be extremely price sensitive.

## 2.4 Responding to ComReg 08/106

### 2.4.1 Pricing principles

ComReg 08/106 states the principles by which prices and cost recovery should be set: cost causation, distribution of benefits, effective competition, cost minimisation, reciprocity and practicability. We agree with these principles.

ComReg further states that causation is not an issue for the split of common and fixed costs between LS and PSTN line rental charges; we similarly agree.

ComReg states that the distribution of benefits principle has been studied 'in section 3 of the TERA report. In section 3, they discuss the issue of possible cost over- or under-recovery. We argue strongly in the next subsection that the *level* of cost recovery should be independent of the *efficient structure* of cost recovery. Further, 'distribution' of benefits, in our opinion (economically speaking) means distribution among all stakeholders (eircom, Cable operators, mobile operators), not just OAOs. Pricing the LS price too low, could lead to negative distributional impacts on other stakeholders, as well as on consumers in the long run. While the 'level' of cost recovery is currently sufficient, we submit that ComReg has not considered the possibility of dynamic changes in the market, whereas voice is becoming more price elastic.

The impact of ComReg's proposal on competition is ambiguous. BB retail offerings using Mobile BB, Cable BB, and full LLU are all competitors of offerings based on bitstream and direct LS products

ComReg argues that the current proposal, zero fixed and common costs gives "strong incentives" to minimise cost. In fact, the power of the incentive scheme is a separate issue, and has not been addressed by ComReg or other consultants working for ComReg. A vague reference to 'overly complex' refunds is also mentioned, which is not explained or founded.

While reciprocity is not an issue, ease of implementation is an issue raised by ComReg, dealt with in section 5 of the consultant's report. As it turns out, neither ComReg nor their consultants has investigated the issue in detail. We have. We estimate elasticities for DSL and voice services and show they are virtually identical. Further, our research shows that a) a Ramsey pricing/global price cap approach means that ComReg can set the overall level of prices and correct incentives to set the structure of charges are likely to exist, and b) the empirical results suggest that the correct level is likely to be very close to a 50/50 sharing rule, and c) Ramsey prices, even in absence of the global price cap approach, can be implemented without detailed cross price elasticity estimates. In addition, Ramsey pricing or variants thereof have been found rather commonly in utility pricing, including ComReg's own decisions relating to other communications markets in Ireland.

ComReg states that the practicality principle is "discussed in *great detail* in section 5 of the TERA report." (our *emphasis*). We submit that this is not the case. The TERA report in fact gives just three bullet points and sentences and then a one word conclusion. "Simplicity: No" (page 42), and then states, "There are a number of difficulties associated with the practical implementation of Ramsey prices." Included is the need to estimate cross price elasticities. In fact a) Indecon has done this, and b) it is not necessary as either a global price cap can be used or cross price elasticities can be omitted (we discuss this later in the empirical and theoretical sections that follow). TERA then discusses the simplicity principle for each of 10 types of pricing. In each case, a single sentence and a single judgment is given. No empirical evidence of the difficulties, no discussion of how or where this might have been used in other jurisdictions, and no citation of international research, under this simple category is given. The section is in our opinion not a discussion of the difficulties of implementing Ramsey in practice.

In fact, Ramsey prices have been implemented in practice for years and across industry including telecoms. Variants of Ramsey pricing are in fact the basis for peak load pricing and pricing of last minute deals and low priced air travel. For example, Ryanair and other low cost carriers charge a high fare for fliers who book late, and high margins on certain items in-flight. They do not run newspaper ads saying "get our 'Ramsey' fares", but the rationale is a simple application of Ramsey principles—charge the lower price in proportion to the inverse of the demand elasticity. It is noteworthy that this is a "win-win", i.e., consumers are better off, because they can get cheap holidays, and low cost airlines can still make profits. Similarly, telecom operators often adopt different prices for day, evening and weekend time periods because the elasticity varies with time of day and day of the week.

Ramsey pricing is the basis of pricing in a number of regulated markets, including electricity prices in general (where peak-load pricing can be seen as the original application of the concept by Boiteux-while working for EdF). The USPS explicitly uses Ramsey pricing principles and estimates the elasticities of products and sets prices according to these elasticities. A more detailed review of these is contained in a later section.

The rest of the discussion is in the conclusions, where it states that the implementation burden (of Ramsey pricing) is 'very high'. Finally, in the conclusions, the recommendation of an "asymmetric method" is made. ComReg, however, has extended this weaker conclusion of its own consultants, to mean, "zero" joint and common costs. Further, the asymmetric method does not seem to include the possibility that the elasticities of demand of each of LS and voice are similar (leading to symmetry being a close approximation of Ramsey pricing, and clearly satisfying the 'ease of implementation' clause).

It seems apparent in our opinion that the TERA decision on simplicity, seemingly the main argument against Ramsey-type pricing, is based primarily on the judgment of ComReg's own consultants. Furthermore, ComReg's analysis appears to implicitly contain a steady state assumption, i.e., they have failed to recognise the declining line base and the probability of demand changes over time. This lack of consideration made for the declining line base and demand changes is likely to put eircom at a distinct disadvantage, as cost recovery may need to be spread over fewer and fewer lines (a BB only product will likely be introduced, but indications are that customers keep voice and BB together).

In conclusion, while the criteria as set out are in theory a sound basis, it appears that ComReg has misinterpreted much of the TERA report, (e.g., the alleged detailed discussion in section 5 about the limitations of Ramsey pricing) solely on the basis of the opinion that the LS rental price should be reduced to zero.

## **2.4.2 Appropriate cost measure**

There is no issue with the basis of costs, as the LLU methodology has addressed this<sup>18</sup>.

## **2.4.3 Review of cost recovery mechanisms in other jurisdictions**

ComReg next gives some discussion of cost recovery in other jurisdictions. The ComReg method is arbitrary and sensitive to the sample selection.

According to ComReg 08/106 and ComReg 08/106a, 10/18 (the EU 15 and the USA, Japan, and Australia) jurisdictions use the incremental costing approach, which ComReg suggest, is the basis for a change of approach in Ireland. Others use a basis of 50/50 or benchmarking, etc. Further, only 4/18 have changed their methodology recently.

However, including Switzerland, Norway, Canada, and NZ, all of which we would argue would likely to be much better comparators to Ireland than say, Australia in the LLU/LS context, changes the results significantly. Now, only 10/22 choose incremental costing, and only 3 out of 22 have essentially allocated zero cost of the fixed, joint and common costs of the local loop to line share (see table). ComReg is thus proposing something that is not used in the majority of the EU and in the OECD's major countries.

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<sup>18</sup> There are of course, ongoing issues with LLU between ComReg and eircom. We merely do not wish to comment on these as they are, in general, separate issues. It may be that these issues do become related to the cost recovery issue, but our main purpose of this paper is to advocate for a particular 'structure' of prices for LS versus PSTN.

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ComReg then asks if respondents, “agree or disagree with the ‘summary’”. Naturally, there are two questions for consultation: 1) does one agree with the summary, and 2) does one agree with the conclusions, methods, etc. The summary of the evidence presented is factual according to our review of the sources. However, more importantly, the conclusions are not based on sound method, as the conclusions might be considered sensitive to the sample selection. We further do not find 3/22, or 3/18 as compelling evidence. So we agree with the evidence but not its interpretation. The conclusion that we make, that zero fixed and common cost allocated to LS is NOT the norm, is conversely, not sensitive to the sample selection.

**Table 2-1: LS Pricing Methodologies in Various Countries**

<b>Country</b>	<b>LS Pricing Methodology</b>
Australia	Incremental
Austria	50%
Belgium	Incremental
Denmark	50%
Finland	50%
France	Incremental
Germany	Incremental
Greece	Incremental
Ireland	50%
Italy	Incremental
Japan	Incremental
Luxembourg	% determined by incumbent
Netherlands	Incremental
New Zealand	No LS
Norway	50%
Portugal	Incremental
Spain	Incremental
Sweden	50%
Switzerland	No LS
UK	Incremental
USA	Some allocation
Canada	Some allocation

Source: ComReg TERA report and Indecon

## 2.4.4 Revue of the assessment of potential pricing methods

ComReg 08/106 and 08/106a claim to have assessed the different methodologies in detail. In fact, there is a list of criterion, consistent with the pricing principles, and little detailed analysis of the actual status of the pricing methods is carried out. No means of weighting or choosing between competing criteria is discussed, and no means of coming to a final conclusion is studied methodically. Apparently, TERA has given a list of pros and cons of each method, and ComReg has decided how to weigh these up based on its own judgment. There are additional details of the TERA report, for example, the FCC method proposed was vacated by the Court on appeal in the USA.

In our opinion, the TERA report is not entirely incorrect to make the conclusions that it does (that the asymmetric method is better, and that incremental costing is OK), however, we believe that the following is a better way of interpreting the results, and is in fact, consistent with TERA's overall logic:

“While the possibility of an asymmetric pricing rule is acceptable, demand implications should not be ignored.” There is also the issue of cost, information, and incentives. We propose that a reasonable method would be to propose a price ratio and an overall pricing level for both PSTN and LS. The firm will likely have an incentive to set the correct structure, while the level can be constrained by the regulation. This is consistent with the Global price cap, and other research<sup>19</sup>.

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<sup>19</sup> According to Armstrong and Vickers (2003), under regulation of a “multiproduct monopolist when the firm has private information about cost or demand conditions. The regulator offers the firm a set of prices from which to choose. When there is private information only about costs, the firm should always have a degree of discretion over its pricing policy”. The Journal of Industrial Economics, Volume 48 Issue 2, Pages 137 - 160 Published Online: 27 Mar 2003.

## 2.4.5 Competition

ComReg reviews the impacts of its decision on competition. According to ComReg 08/106's figure 3, the greatest increases over the last few quarters for broadband (BB) subscribers has come from mobile BB (which also has the second highest share of the market at 21%), Cable BB, FWA, and FTTH together make up about 21% of BB subscribers (about equal in share to Mobile BB-21%), and have been seemingly increasing faster than DSL in the last quarter or two (the exact numbers are not given). Together, non DSL BB makes up about 42% of the market, compared to 58% for DSL. Further, according to ComReg, under the current pricing regime, LS lines have been increasing rapidly in Ireland (page 9 tells of 120% growth).

ComReg then makes the following conclusion: "as long as the total cost of the access network *in aggregate* is recovered based on costing principles not unfavourable to platform competition it would appear there can be *no distortion* of inter-platform competition..." This is apparently based on a legal decision. We can only comment on the economic logic.

To consider the economic logic, consider the following thought experiment. The incremental cost of say, Mobile BB is not zero – but socially it is the lower cost service for voice and BB. Clearly, if the lower cost service (on the whole) is say, Mobile BB, because mobile is increasing fastest, soon will have/has more voice customers to spread fixed costs, etc, and if the line share price is set to zero, investment in the lower cost service (mobile BB) could be stymied and held back, thus raising the overall cost to society.

Allowing for complementarities between voice and BB also shows the flaws in the economic logic. Suppose voice and BB are compliments in demand, but higher voice prices cause people to leave voice line rental. The loss of voice lines could easily *reduce* the overall level of BB subscribers, even if a boost to LS is given by a low price.

Overall we believe that ComReg has not studied the case of the impact on alternative investment in Ireland in sufficient detail to justify its current position and reverse the 50/50 rule.

## **3 Detailed discussion of Ramsey and incremental pricing approaches**

### **3.1 Introduction**

This section discusses the theory and practice of Ramsey pricing more generally and in the context of the current consultation.

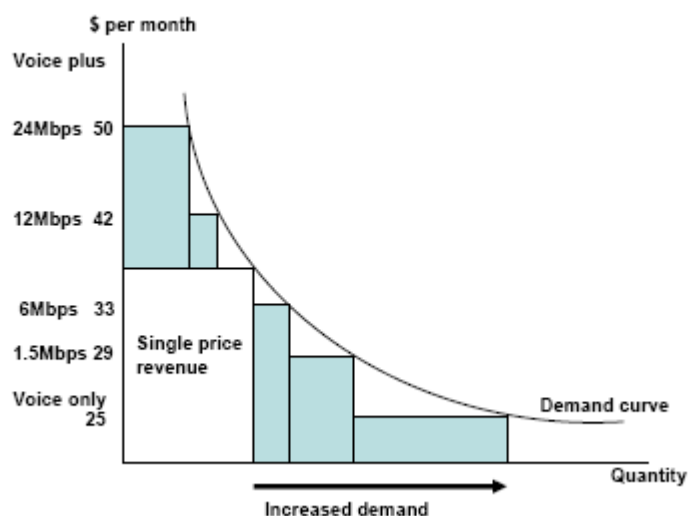
ComReg/TERA are of the view that Ramsey Pricing, while attractive in principle, is little used in telecoms regulation (including in LS) owing to perceived practical difficulties in terms of implementation. The TERA report states that while regulators could try to “approximately implement [Ramsey pricing] there have been no known attempts to do so” (p. 41). (We would disagree. ComReg, for example, in setting postal prices, has explicitly considered demand elasticities and the relative merits of recovery of fixed and common costs with regards to minimising the impact on demand). Furthermore, section 2.4.3 above and Table 2-1, and our review of Ramsey pricing in the following subsections of this section 3 (e.g., see section 3.6 that follows), indicate that Ramsey Pricing or similar methods have, in fact, been implemented by regulators in other jurisdictions.

In this section, we argue that Ramsey pricing is not insurmountable in the context of cost allocation in LS, provided price elasticities of demand can be calculated. Ramsey pricing is also generally accepted as beneficial to the consumer. Subsequently in this document, we report the results of econometric estimates in respect of own- and cross-price elasticities of demand for retail and wholesale broadband (i.e., DSL) and voice (i.e., PSTN) services (for eircom).

### **3.2 Basics of Ramsey pricing**

The optimal economic solution to efficient recovery of fixed costs, as argued by both established institutions and leading economists, is Ramsey pricing. This also can be applied to joint and common costs. This sets mark-ups based on what retail customers are willing to pay. A key idea here is that incumbents should be sufficiently compensated for their initial investment, but this is independent of the structure of the charges.

Figure 1: Ramsey Pricing in Telecommunications



Source: Goldilocks Pricing for Broadband [www.deridder.com/au/files/goldilocks.pdf](http://www.deridder.com/au/files/goldilocks.pdf)

The concept of Ramsey pricing is relevant in situations where there are large fixed costs of production so that marginal costs of producing another unit of output are significantly lower than the average. Eircom is a case in point of this particular situation. A key element is that fixed costs are allocated across products in inverse proportion to the products' elasticities of demand. In this way, Ramsey pricing allocates common costs to customers based in proportion to the elasticity of demand of the various classes of services.

### 3.3 Importance of Demand-Side Considerations in Regulatory Price-Setting

Central to this consultation response document is the view that Ramsey pricing is not as difficult as is claimed by ComReg (but also by many economists) We provide arguments in support of this view based on consideration of recent economic research on Ramsey pricing and on our own analysis illustrating how Ramsey pricing may be applied in practice (giving rise subsequently to new estimates of own- and cross-price elasticities of retail and wholesale broadband and voice services).

Generally speaking, application of Ramsey pricing involves consideration of the demand-side of the market and, by way of background to our arguments showing that Ramsey pricing is not as difficult as traditionally thought, we first highlight the importance of incorporating demand-side considerations in regulation (i.e. as a general principle of good regulatory practice).

The very name of Boiteux, in the Ramsey-Boiteux pricing name is based on the application of the Ramsey principle to peak-load pricing in the electricity sector. Peak load pricing and peak charging have been factors in telecoms pricing for decades (See for example, Wilson, 1982, 'Nonlinear Pricing').

The importance of demand-side considerations in price setting in regulated markets is well-made in Hausman and Sidak (2007, p. 10):<sup>20</sup>

“In competitive markets, firms set price based on cost conditions, demand conditions, and competitive conditions. Regulators attempt to base prices on only the first of these three factors...demand elasticities are almost never used. Yet in competitive markets demand elasticities are an important component of pricing decisions in a multi-product situation. Thus, regulators do not meet their goal of setting regulated prices in a manner similar to that of a competitive market. Furthermore, they can cause billions of dollars per year of losses in economic efficiency and consumer welfare. Instead of using inherently arbitrary allocation procedures, regulators could improve the outcome of the regulatory process either by taking account of demand and competitive conditions in setting regulated prices or adopting procedures such as global price caps, which will lead the regulated utility to take account of demand and competitive conditions”.

Hausman and Sidak, two leading economists in the regulatory field, proceed in their study to examine the implications of regulatory models based on cost but without demand-side considerations. They begin with cost-based regulation. In a simple one-period and one-good production model with constant returns to scale, a partial equilibrium analysis demonstrates that the competitive price does not depend on demand. However, this simple model is unlikely to hold up in practice – for example, telephone and wireless networks have a very large proportion of fixed, common and sunk costs, so that marginal and average cost are not independent of the quantity produced (as in the simple model).

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<sup>20</sup> ‘Telecommunications Regulation: Current Approaches with the End in Sight’, Oct 2007. Available at: [http://www.nber.org/books\\_in\\_progress/econ-reg/hausman-sidak10-5-07.pdf](http://www.nber.org/books_in_progress/econ-reg/hausman-sidak10-5-07.pdf).

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Hausman and Sidak (2007) show that to correctly set prices independent of demand, four assumptions are needed, the two most important of which are no economies of scale and economies of scope. Both assumptions are clearly violated in all telecommunications markets/networks, and also in the current context of LS and PSTN (the zero cost incremental cost and the stand-alone cost above zero implies economies of scope.)

According to Hausman and Sidak (2007, pp. 16-17):

Thus, our evaluation is that modern telecommunications differ in many significant and quantitatively important ways from the necessary conditions for price to be independent of demand. Economies of scale and economies of scope are universally recognized to be important economic characteristics of modern telecommunications networks. The regulatory attempt to set prices as if they were the outcome of a competitive process but to ignore the importance of demand factors and competitive conditions leads to what we call the *regulatory fallacy* ... Economic theory has developed precise condition[s] when price is independent of demand, and they do not hold, even as an approximation, in telecommunications. Thus regulators are acting on an erroneous belief that, with competition, price equals cost, independent of demand. This erroneous belief leads directly to the resulting regulatory fallacy. The consequent use of arbitrary allocations and markups to regulated prices to take account of fixed and common costs – which are *exactly the costs that arise from economies of scale and scope* – leads to significant consumers harm. If regulators instead took account of demand factors in setting regulated prices, economic efficiency and consumer welfare could be increased significantly”.

### 3.4 Ramsey Pricing and its Advantages in Theory and Practice

Ramsey or Ramsey-Boiteux pricing is a regulatory rule concerning what price a monopolist should set in order to maximize social welfare subject to a constraint on profit (e.g., non-negative or zero profit). For the case of a single-product monopolist, the price-cost margin is inversely proportional to the own-price elasticity of demand ( $e$ ) of the good or service: the more elastic the demand, the smaller the price-cost margin. In a multi-product natural monopoly setting, Ramsey-Boiteux pricing implies that the price-cost margin of each product is also inversely proportional to the corresponding own-price elasticity of demand for each product but now the price-cost margin is lower than implied by monopoly because the inverse own-price elasticity of

demand is multiplied by a constant lower than 1 (the Ramsey number,  $\alpha$ ).

In other words, the effect of Ramsey pricing is to reduce the monopoly price-cost margin uniformly for all units so that only the required revenue is obtained by the firm.

Ramsey pricing is a form of price discrimination in which segments of the market with a more elastic demand pay less and this is efficiency-enhancing since it means expanding the range of services provided. According to the OECD (2004, p. 28):<sup>21</sup>

“For example, rather than charge a single price (equal to average cost) for a service, it is often more efficient, if feasible, to charge a two-part price, with a fixed charge for “access” (not to be confused with access to essential inputs) and a separate charge for “usage”. Ramsey pricing can then be used to set the price for each of these two new services. It is important to take into account the fact that these two services will usually be complements – a reduction in the usage charge may increase the number of customers wishing to sign up for service. Depending on the relative elasticities, it may, therefore, make sense to charge above marginal cost for usage in order to lower the fixed charge, to encourage greater demand for the overall service”.

### **3.4.1 Ramsey pricing maximises social and consumer welfare**

The welfare advantages of Ramsey pricing are well recognised and include:

- Incorporation of demand-side considerations (and thus overcomes the criticism made by Hausman and Sidak of other regulatory mechanisms that ignore demand-side considerations);
- Maximisation of social welfare (including consumer welfare);
- Cost recovery for the incumbent or network owner;
- Avoidance of cost over-recovery by the incumbent or network owner;
- Encouragement of efficient investment in infrastructure.

In the context of access pricing, where a firm produces two products (namely access to competitors and retail telephony services to final consumers,

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<sup>21</sup> OECD Report on Access Pricing (2004): <http://www.oecd.org/dataoecd/26/6/27767944.pdf>



Ramsey pricing leads to the following results: first, all prices should contribute to the fixed cost recovery problem; and (ii) the optimal contribution depends on the price elasticity of demand for each product – the higher the price elasticity, the lower the contribution.<sup>22</sup>

In a recent paper, de Ridder (2008) looks at the billion dollar investment in Australia regarding the building of a fibre-to-the-node (FTTN) network, which will require prices high enough to justify the investment but low enough to make services affordable to end-users.<sup>23</sup>

Whoever builds the FTTN, the key issue will be wholesale pricing of bitstream access and its impact on affordable broadband access and competition. De Ridder (2008) considers a range of access pricing options for the FTTN against the following criteria:

- It must provide a return on a multi-billion dollar investment;
- It must ensure that broadband access remains affordable to end-users;
- It must provide affordable access for wholesale customers;
- It must not foreclose alternative access investment.

The current methodology for regulating access pricing in Australia, de Ridder (2008) notes, is based on total service long run incremental cost plus a contribution to fixed and common costs (TSLRIC+). De Ridder shows that while TSLRIC+ does not foreclose alternative access investment, it does not satisfy the first of the above criteria and it is not known whether TSLRIC+ satisfies the second and third criteria. On the other hand, Ramsey pricing is shown by de Ridder (2008) to satisfy all four criteria, like the ECPR (efficient component pricing rule) and retail minus pricing (which are variants of Ramsey pricing). According to de Ridder (2008, p. 13):

“All of the approaches that reference retail prices (i.e. ECPR, Retail-Minus and Ramsey) are virtually equivalent and meet all criteria”.

In choosing between Ramsey pricing and ECPR, which is closely related to Ramsey pricing,<sup>24</sup> noteworthy is Vickers (1997) points out that ‘Ramsey

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<sup>22</sup> K. Kotakorpi (2002), ‘Access Pricing and Competition in Telecommunications’, Government Institute for Economic Research, Finland

<sup>23</sup> De Ridder (2008), ‘Goldilocks Pricing for Broadband’, *Telecommunications Journal of Australia*, Vol. 58 (No. 1), pp. 1-13

<sup>24</sup> Larson and Lehman (1997), ‘Essentiality, Efficiency and the Efficient Component-Pricing Rule, Larson and Lehman (1997) *Journal of Regulatory Economics*

principals give optimal retail and access prices when there are fixed costs to be recovered'.<sup>25</sup>

According to a World Bank report<sup>26</sup>, the basis for access pricing above marginal cost in general is that 'deficits are socially costly and the charge performs as a tax used to raise money that repays the deficit'. The chart below, which is provided by the World Bank, outlines access charges with vertical integration.

**Table 2: Access charges with vertical integration**

Basic Case	Access charge	Potential problems	Eventual remedies
First best	Marginal cost	Require lump sums, otherwise fixed cost not covered	Tariff rebalancing USO funds
Second best	Ramsey	Informational content may not be sustainable	Price cap
Productive efficiency	ECPR	Partial rule	

Source: Excerpt from World Bank Report (see footnote 1)

Generally, regulators will need good reasons to depart from efficient outcomes, or outcomes which replicate those which might be expected in a competitive market, when setting prices<sup>27</sup>. The fact that Ramsey pricing may be difficult to implement in practice is not a good reason or indeed any reason at all for NRAs to avoid its obligations.

<sup>25</sup> Vickers, J. (1997) 'Regulation, Competition and the Structure of Prices', *Oxford Review of Economic Policy*, Vol. 13(1), pp. 15-26.

<sup>26</sup> Valletti & Estache, 2008, 'The Theory of Access Pricing: An Overview for Infrastructure Regulators', Valletti (2008), London School of Economics and The World Bank Institute: <http://www.worldbank.org/html/dec/Publications/Workpapers/wps2000series/wps2097/wps2097.pdf>

<sup>27</sup> See Article 8(2)c of the Framework Directive (Reference taken from a paper by Vodafone): [http://www.erg.eu.int/doc/publications/call\\_input\\_lric/vodafone\\_final.doc](http://www.erg.eu.int/doc/publications/call_input_lric/vodafone_final.doc)

The Ramsey pricing method is advocated for by many prominent regulatory economists in the literature. Perhaps, most notably, Laffont and Tirole (1994)<sup>28</sup> who make arguments in favour of the pricing method. In fact, Laffont and Tirole (2000) explicitly state, “we find it surprising that regulators who routinely design price caps dismiss offhand Ramsey Pricing as being informationally infeasible.” While ComReg quotes Laffont and Tirole as supporting their view, their quotes are seemingly so selective as to be misleading.

Contrary to the argument of some regulators Ramsey pricing promotes efficient allocation of resources and is often cited as the most equitable methodology for pricing in access markets. For example, Whalley (et al) make the point that ‘Ramsey pricing aims for global efficiency through ensuring that all necessary goods are produced’. The paper also outlines an equitable basis for the approach arguing that ‘because Ramsey pricing takes into account demand elasticity it is also generally perceived as a social correction as well’.<sup>29</sup> Furthermore, Sherman (2003)<sup>30</sup> and others continue this line of thinking noting that Ramsey prices will ‘not reflect the knife edge conditions that are implicit in the use of the ECPR or MECPR’ and perhaps even more importantly that they ‘allow achievement of the greatest welfare even when demands are not infinitely elastic’.

Building on the work of Laffont and Tirole (1994), a more recent paper by Vogelsang and Finsinger suggests that Ramsey prices can be even more effective when employed through a price cap system.<sup>31</sup> The cap methodology ensures an equitable transfer of welfare gains among customers.

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<sup>28</sup> Laffont and Tirole (1994), *Access Pricing and Competition*, *European Economic Review*

<sup>29</sup> Whalley, Verhoest and Steinmueller, ‘Asymmetric Pricing of Unbundled Infrastructure Components: Effective Competition versus Economic Efficiency’

<sup>30</sup> Sherman, R., ‘Restructuring Industries: The Carrot and the Stick’, Department of Economics, University of Houston

<sup>31</sup> Vogelsang and Finsinger (1978), ‘Regulatory Adjustment process for optimal pricing by multi-product monopoly firms’

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## 3.5 Recent Economic Research on Ramsey Pricing

In addition to the literature, the concept of Ramsey pricing has also been advocated by prominent institutions such as the Organisation for Economic Co-Operation and Development (OECD). In its 2004 Access Pricing Report, the OECD state that ‘if marginal cost pricing yields insufficient revenue to the monopolist then both access and final prices must be raised above marginal cost according to the Ramsey Formula’.<sup>32</sup>

Moreover, Ofcom<sup>33</sup> (in a 2007 report) shows its support of pricing customers on their willingness to pay for vertically integrated NGN access providers:

‘We consider that it is important to allow a degree of flexibility for investors to price access. The reason for this is that the total value derived from next generation access networks is the sum of different valuations by different end users’.

### 3.5.1 Ramsey pricing is not complex

ComReg makes the argument that the Ramsey pricing approach is overcomplicated. We suggest that this is not the case. As a first point, there is considerable recent literature to suggest (as outlined in the literature section above) that Ramsey pricing would provide an optimal welfare outcome and therefore its implementation may be worthwhile.

Secondly, it appears to be the case that aversion to Ramsey pricing is often based on the difficulty of obtaining data and estimating customer elasticity’s. The idea that complexity should always obstruct implementation has been refuted by some economists in the recent literature. For example, Laffont and Tirole (1996) respond to the criticism that Ramsey pricing requires a significant amount of demand information arguing that an appropriate global price cap can induce Ramsey pricing in a decentralized manner.<sup>34</sup>

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<sup>32</sup> OECD Access Pricing (2004): <http://www.oecd.org/dataoecd/26/6/27767944.pdf>

<sup>33</sup> Ofcom Report: Future Broadband, Policy Approach to Next Generation Access, Office of Communications (2007)

<sup>34</sup> Laffont and Tirole (1994), ‘Creating Competition through Interconnection: Theory and Practice’, *Journal of Regulatory Economics*, 10, 227-256.

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Furthermore, a report by the World Bank not only argues that complexity is not a sufficient argument by regulators but that they have an obligation to overcome these informational problems. The report acknowledges that Ramsey pricing requires 'a good deal of information' but goes on to make the point that 'difficulty, however, does not imply infeasibility'<sup>35</sup>. Finally, the report notes that while elasticity's may sometimes be difficult to forecast that the 'patterns of demand are rather standard and predictable so that the regulators could and should try to produce such estimates.'

### 3.5.2 ECPR and Ramsey Pricing

A comparable technique to Ramsey pricing is the Efficient Component-Pricing Rule (ECPR). The idea of the rule is that access price equals the incumbents opportunity cost on competitive segment.

Luis (et al) point out that 'ECPR is popular among regulators'<sup>36</sup>. In addition to this Dewenter identifies ECPR as one of the 'most prominent' access pricing methods.<sup>37</sup> ECPR has been used in New Zealand Supreme Court (1994), US Telecommunications Act (1996) and by Oftel in the UK (however it was abandoned in 1997).

Armstrong (1996 et al)<sup>38</sup> finds that the Ramsey approach to access pricing developed by Laffont and Tirole (1994) is closely related to the ECPR approach provided opportunity cost is correctly interpreted. The basis behind this argument is that the opportunity cost can be a vague concept and this makes it difficult to estimate. Armstrong goes even further than this, concluding from their analysis that 'ECPR does not offer any advantage over the complex Ramsey pricing rule'. In addition to this, Larson (1997 et al) find that ECPR can be derived as a special case of a Ramsey pricing rule.<sup>39</sup>

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<sup>35</sup> Valletti & Estache, 1998, 'The Theory of access pricing: an overview for infrastructure regulators', London School of Economics and The World Bank Institute:  
<http://www.worldbank.org/html/dec/Publications/Workpapers/wps2000series/wps2097/wps2097.pdf>

<sup>36</sup> Cabal, M.B, 2000, 'Introduction to Industrial Organisation'

<sup>37</sup> Dewenter and Haucap (2006), 'Access Pricing Theory and Practice'

<sup>38</sup> Armstrong, Doyle and Vickers, (1996), 'The Access Pricing Problem: a Synthesis', *Journal of Industrial Economics*, 2, 1996, pp. 131 - 50

<sup>39</sup> Larson and Lehman (1997), Essentiality, Efficiency and the Efficient Component-Pricing Rule, *Journal of Regulatory Economics*

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In fact, Laffont and Tirole (1994)<sup>40</sup> and Larsen<sup>41</sup> show that EPCR and Ramsey pricing are equivalent for a certain set of conditions. These conditions are: (1) The downstream services of incumbent and entrant(s) are perfect substitutes (2) The entrants have no market power (Bertrand competition) downstream (3) The downstream industry produces at constant returns to scale (4) The benchmark pricing rule is marginal cost pricing (5) There is no bypass in the upstream market.

However, there are multiple strands in the recent literature to suggest that ECPR is a less effective way of achieving welfare gains than Ramsey pricing. For example, Yannelis (et al) find that simple ECPR does not offer any advantage over Ramsey pricing.<sup>42</sup>

The main perceived disadvantage of Ramsey pricing is that it is difficult to apply for regulatory purposes in practice. A common economic model of a regulated market is that of a mixed oligopoly, in which a regulated incumbent firm competes against one or more unregulated firms offering substitute or complementary products or services (OAOs). Traditional economic models show that above-marginal-cost pricing in the competing sector would require data facilitating estimation of own and cross price elasticities of demand to establish Ramsey prices different than would have existed in the absence of such competition.

A different view is taken by Braeutigam (1979).<sup>43</sup> In particular, they show that the simple inverse elasticity rule to pricing applies when a competitive fringe prices at marginal cost. In this situation, the regulator needs only cost and own price demand elasticity data for the regulated firm in order to establish globally optimal Ramsey prices.

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<sup>40</sup> Laffont and Tirole (1994), 'Access Pricing and Competition', *Eur. Econ. Rev.* 38, pp. 1673-710

<sup>41</sup> Larsen and Alexander (1995), 'Interconnection and Access Pricing: A Derivation of the Efficient-Component Pricing Rule', mimeo, South-western Bell Telephone Co

<sup>42</sup> Yannelis, 'On Access Pricing with Network Externalities', Department of Economics, University of Piraeus

<sup>43</sup> Braeutigam, R. (1979) 'Optimal Pricing with Intermodal Competition', *American Economic Review*, Vol. 69, pp. 38-49

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Building on these earlier insights, a recent paper by Miller (2007)<sup>44</sup> establishes a more general condition showing that the same inverse elasticity rule can apply with above-marginal-cost pricing when rivals are Cournot competitors and zero long run profits prevail. Cournot competition and zero profits might be considered appropriate where there is relatively little product differentiation within the competitive sector and there are (potentially) a large number of rivals caused by weak scale effects relative to demand. The regulated firm's product is differentiated from rivals as a whole, possibly because of reputation or another factor (e.g. reliability). Cournot competition occurs in quantities (i.e., competitors seek to maximize their profits on the basis of output setting decisions and the Cournot price is above the marginal cost but lower than the monopoly price, the mark-up of which is given by the inverse of the market price elasticity of demand,  $1/e$ ).

The particular model presented by Millar (2007) assumes that the regulated firm offers a non-competitive and competitive product where the firm acts as a Stackelberg price leader. Rival firms take any changes in the competitive product rate as given when adjusting their own output according to Cournot assumptions. Free entry and exit conditions exist and therefore a zero-profit equilibrium is re-established (once new firms enter or existing firms exit). Within this framework, Millar shows that deviations from the simple inverse elasticity rule for Ramsey rates require long run changes to the competitive sector rate. It is further shown that long run price changes are absent with linear demand and therefore the inverse elasticity rule applies in this instance. Therefore situations where limited (regulated firm only) data requirements for Ramsey pricing are considered appropriate might be broader than previously thought.

The environment in Millar's (2007) model is a two-stage game where the regulator first sets prices for the regulated firm's two products subject to a break-even constraint, and then the price and the number of firms in the competitive sector adjusts re-establishing equilibrium. Each rival has the same cost structure and therefore identical (zero) profits given the common rate charged. Millar shows that, under linear demand, the regulator needs to use only the regulated firm's own price demand elasticities for the two products to determine the optimal (Ramsey) rates for the prices of goods 1 and 2 according to the simple inverse elasticity rule.

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<sup>44</sup> Miller, W. C., (2007), 'Ramsey pricing with long run competition', *Economics Bulletin*, Vol. 12, No. 34, pp. 1-5

This is, Ramsey pricing would imply in equilibrium that:

**Equation 10**

$$\frac{e_2(p_2 - c_2)}{p_2} - \frac{e_1(p_1 - c_1)}{p_1} = 0$$

Where  $p_i$ ,  $e_i$  and  $c_i$  are the prices, price elasticities of demand and (constant) marginal costs respectively of the two goods  $i$  ( $i = 1, 2$ ). Re-arranging this condition means that the ratio of the regulated firm's price-cost margins is equal to the ratio of its own price elasticities of demand, *viz.*:

**Equation 11**

$$\frac{pcm_1}{pcm_2} = \frac{e_2}{e_1} \quad [pcm_i = \frac{(p_i - c_i)}{p_i}]$$

Thus, all that would be required to apply Ramsey pricing in this case would include estimation of the regulated firm's own price elasticities for the competitive and non-competitive products (i.e. estimation of eircom's own-price elasticities of demand for wholesale and retail DSL/broadband and PTSN/voice services).

We consider that Millar's recent model may be applicable in the context of this consultation exercise because of the following factors:

- eircom's position may be that of a leader in the competitive market (retail PTSN);
- It would be plausible to assume that competitors would take eircom's output in the competitive market as given in deciding what to provide in the competitive market;
- Free entry and exit conditions exist in the competitive market;
- Linear demand represents a workable and simple conceptualization of eircom's demand in both markets.



## 3.6 Examples of Ramsey Pricing

In addition to the de Ridder (2008) study above, another example of the use of the Ramsey pricing approach is in Chinese electricity tariffs, where high rates for commercial customers allow subsidized consumption in residential sectors.<sup>45</sup> In this study, econometric methodologies are applied to estimate the price elasticity's of demand for residential and industrial consumers based on the data of 22 provinces in China during the 2003-2005 period. The paper concludes that the Ramsey pricing scheme could improve social welfare.

In a study by Cuthbertson and Dobbs (1996),<sup>46</sup> a case study of the UK letters business is used to illustrate the approach of Ramsey pricing. The study's sensitivity analysis suggests that 'Ramsey prices are fairly robust to variations in demand and costs parameters, which implies that existing prices at the time of the study were probably not far short of being Ramsey optimal'.

According to a recent Vodafone report,<sup>47</sup> the existing guidelines on LRIC cost modelling<sup>48</sup> note that common costs are most efficiently recovered (or said another way, distortion is minimised) through the application of Ramsey pricing. They argue that any revision to these guidelines should be designed to assist regulators in operating Ramsey pricing in the presence of significant fixed and common costs.

Other examples of Ramsey pricing which have been identified in the literature include its application in US rail, the United States Postal Service (USPS) and EdF electricity pricing. In the case of US rail, Ramsey pricing was employed up until an adequate profit level was attained.<sup>49</sup> Additionally, both

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<sup>45</sup> Qi Zhang and Que (2008), 'An application of ramsey pricing in solving the cross-subsidies in Chinese electricity tariffs', Electricity Utility Deregulation and Restructuring the Power Technologies: <http://ieeexplore.ieee.org/Xplore/login.jsp?url=/iel5/4511470/4523365/04523447.pdf?temp=x>.

<sup>46</sup> 'A robust Methodology For Ramsey Pricing With an Application to UK Postal Services', Cuthbertson and Dobbs (1996), *The Journal of Industrial Economics*, Volume XLIV, 0022-1821 No.3: <http://www.jstor.org/stable/2950495>.

<sup>47</sup> A report by Vodafone: [http://www.erg.eu.int/doc/publications/call\\_input\\_lrlic/vodafone\\_final.doc](http://www.erg.eu.int/doc/publications/call_input_lrlic/vodafone_final.doc)

<sup>48</sup> LRIC model developed by OFTEL (reference from Vodafone Report above)

<sup>49</sup> O'Connor, T., (2005), 'Progress Since the Staggers Rail Act of 1980: A Retrospective Review and a Win-Win Action Plan For Consideration by the Surface Transportation Board STB Ex Parte No. 658', The 25<sup>th</sup> Anniversary of The Staggers Rail Act of 1980, A Review and a Look Ahead

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Scott (1986)<sup>50</sup> and Gomez-Ibanez (1999)<sup>51</sup> cite examples of the use of Ramsey pricing in the USPS. Finally, Newbery (2005)<sup>52</sup> observes the implementation of Ramsey pricing in EdF Electricity pricing.

### 3.6.1      **Capacity Pricing**<sup>53</sup>

Another form of price discrimination that is comparable to Ramsey pricing is known as capacity<sup>54</sup> pricing or nonlinear pricing. Wilson (1982) devotes a whole chapter to Ramsey pricing and shows how two part, block declining, and quantity discounts can be seen as special cases of Ramsey prices. This is a two-part wholesale access price where the fixed charge recovers fixed costs and the usage component recovers marginal costs. Mitchell (1978) contrasts the welfare effects of a flat monthly rate with measured service pricing under an optimal two part tariff with an access line charge and a per call charge.<sup>55</sup>

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<sup>50</sup> Scott, F., (1986), 'Assessing USA Postal Ratemaking: An Application of Ramsey Pricing Prices', *Journal of Industrial Economics*, Vol. 34 pp.279-90

<sup>51</sup> Gomez-Ibanez, J., (et al), (1999), 'Essays in Transportation Economics and Policy: A Handbook in Honour of John. R Meyer'

<sup>52</sup> Newbery, D., (2005), 'Market Design', Paper presented at Conference, Implementing the Internal Market of Electricity: Proposals and Time-Tables, pg 20

<sup>53</sup> This discussion of capacity pricing is in the very general economic sense, and is not related to other discussions/dealing Eircom has had with respect to particular capacity pricing proposals for Wholesale Bitstream Access. The basic concept here is that capacity pricing is a special case of Ramsey pricing.

<sup>54</sup> We note that Indecon uses the term 'capacity pricing' in the pure economic sense. We understand eircom have had discussions with ComReg with regards to a particular application of capacity pricing, and this is a different issue.

<sup>55</sup> Bridger M., (1978), 'Optimal Pricing of Local Telephone Service: A Survey', *American Economic Review* 68 (Sept.1978), 517-537.

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Figure 2: Capacity Pricing

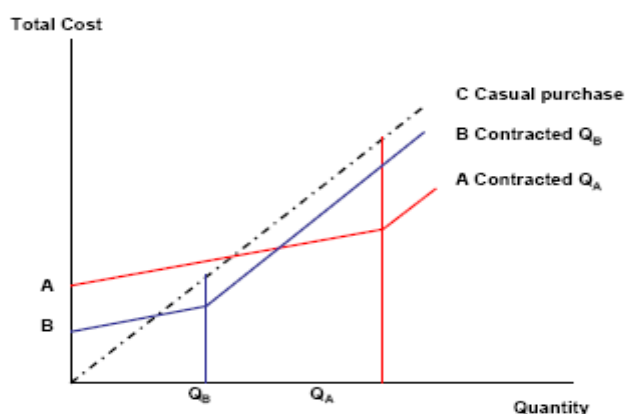


Figure 3: Capacity Pricing

Source: Goldilocks Pricing for Broadband: [www.deridder.com/au/files/goldilocks.pdf](http://www.deridder.com/au/files/goldilocks.pdf)

The benefits of capacity pricing are also recognised by the OECD. It states that 'capacity based pricing allows downstream firms to, effectively, purchase a share of the essential facility, scaled down, to match the requirements of the downstream firm, but with the same cost drivers as are faced by the incumbent operator'.

So in general, capacity pricing<sup>56</sup> is a special case of Ramsey pricing. One finds many examples of capacity pricing in practice, from airlines, to electric utilities, to telecoms and communications (peak charging for voice was an element in some of the earliest pricing schemes under the Bell system in the USA).

The table summarizes some of the different methods of pricing under different headings. Ramsey pricing is identified as being a positive pricing approach for all categories.

<sup>56</sup> We note that Indecon uses the term 'capacity pricing' in the pure economic sense. We understand ericom have had discussions with ComReg with regards to a particular application of capacity pricing, and this is a different issue.

Table 3 – Various Pricing Methods

	Return on investment	Affordable to end users	Affordable to open access	Does not foreclose alternative investment
TSLRIC+	No	Do not know	Do not know	Yes
+WACC	Yes	Probably not	Probably not	Yes
Holiday	Yes	Yes	No	Yes(not FANCO)
ECPR	Yes	Yes	Yes	Yes
Retail-Minus	Yes	Yes	Yes	Yes
Forbearance	Yes	Yes	Yes	Yes
Ramsey	Yes	Yes	Yes	Yes
Capacity	Do not know	Do not know	Do not know	Yes
Anchor	Yes	Yes	Yes	Yes

Source: The above table summarizes the analysis of the paper 'Goldilocks pricing for broadband', Ridder (2008).

### 3.7 Summary

In summary, the concept of Ramsey pricing is very general and many of the current practices in utility pricing and telecoms pricing around the world and for years have in fact been incorporating Ramsey pricing, demand side conditions, and similar considerations. Many common forms of pricing practices in utilities are based on Ramsey principles.

The argument that it is “too complex” is refuted by many economists and practitioners. Further, it is not clear how ComReg judged the scale of complexity is measured versus the benefits of Ramsey pricing.

## 4 Empirical estimates of elasticities in telecoms

### 4.1 Introduction

This sub-section presents our estimates of the price elasticity of demand for various Eircom products using various levels of aggregation (e.g., Broadband, PSTN rental, retail and wholesale).

The main point of the results is that empirical estimation of elasticities suggests that the elasticities of BB and PSTN services are similar in Ireland.

### 4.2 Elasticity estimates using AIDS model

The AIDS estimation approach is more sophisticated from other approaches to elasticity estimation, such as the structural time-series approach, in that it imposes restrictions on the data from consumer demand theory. The imposition of restrictions is useful when data are volatile, when there are inter-related demands, and/or when the investigator has prior knowledge of relationships among variables that should hold. Further, these restrictions reduce the number of free parameters to estimate, and thus increase the numbers of degrees of freedom of the estimates, and thus reduce the variance of the estimates.

We therefore estimate a particular class of demand models, known as the almost ideal demand system (AIDS). The AIDS model, developed by Deaton and Muellbauer (1980)<sup>57</sup> is a system of equations approach to demand estimation. Total customer expenditures on particular products are broken down into a system of equations – namely, a (log) total expenditure equation ( $\ln X$ ), and expenditure-share equations ( $s_i$ ). The system of share-equations is then estimated simultaneously as an iterative seemingly unrelated regression (iSUR) system with constraints. The AIDS model with the linearized share equations can be written as follows:

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<sup>57</sup> Deaton, Angus S & Muellbauer, John, 1980. "An Almost Ideal Demand System," *American Economic Review*, American Economic Association, vol. 70(3), pages 312-26.

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**Equation 10:**

$$\ln x = a + \sum_i \alpha_i \ln p_i + \frac{1}{2} \sum_j \sum_i \gamma_{ij} \ln p_i \ln p_j + b \prod_i p_i^{\beta_i}$$

$$s_i = \alpha_i + \sum_j \gamma_{ij} \ln p_i \ln p_j + \beta_i \ln \left( \frac{x}{P^*} \right)$$

Source: Deaton and Muellbauer 1980.

In the equations above;  $\ln$  is the natural log; the  $p_i$  are the prices of the products, (e.g., LS, Bitstream, full-LLU, etc.), the  $s_i$  is the share in revenue,  $x$  is total revenue (e.g., total LLU access revenue, total revenue),  $P^*$  is a general price index, and the lowercase Greek letters are the parameters to be estimated.

The system is made more tractable (reducing the number of free parameters to be estimated) by imposing restrictions within and across equations, including symmetry, homogeneity, and adding up (budget shares must sum to one) based on consumer theory. The restrictions are:

**Equation 11**

$$\sum_j \gamma_{ij} = 0, \sum_i \beta_i = 0, \sum_i \gamma_{ij} = 0, \gamma_{ij} = \gamma_{ji}, \sum_i \alpha_i = 1$$

Source: Deaton and Muellbauer 1980.

From the above system of equations and parameter estimates, the own and cross-price elasticity estimates can be derived.

**Equation 12**

$$\varepsilon_{ii} = \frac{1}{s_i} \left( \gamma_{ij} + s_i^b \beta_i \right) - 1$$

$$\varepsilon_{ij} = \frac{1}{s_i} \left( \gamma_{ij} + s_j^b \beta_i \right)$$

Source: Deaton and Muellbauer 1980.

## **Data**

Implementation of the model above requires data and observations on telecoms LS, Bitstream, PSTN rental, and LLU volumes, prices, revenues, and a general price index over time (in fact, only 2 out of three, volumes, prices and revenues are needed as one becomes endogenous to the other).

Data on prices for PSTN and broadband services were provided by eircom. Quantity are 'lines' and prices are standard price offerings. Data were monthly from 2004 to the present. Data were split into retail and wholesale offerings, and broadband wholesale offerings were split into speeds. There were also splits by business and home. We aggregated over speed classes, and kept the model as residential only, as this represented a very large portion of users.

Aggregate price indices, such as the CPI (monthly) and services CPI were taken directly from the CSO website.

## **Results**

We implemented the quadratic or QUAIDS estimation system for a system of share equations generated for the model consisting of four products: retail and wholesale PSTN rental, and retail and wholesale broadband rental. The estimation for the system used a quadratic AIDS (or QUAIDS) approximation using the computer statistical software package STATA, and programming the required equations and within and across equation restrictions was part of a subroutine provided by STATA researchers available with the software license. The results yielded include terms that can calculate own-price and cross-price elasticity estimates and their components for the estimation of a four-product system

To estimate the QUAIDS model discussed above, products were aggregated into DSL and PSTN groupings, using Eircom data on Retail and Wholesale Line bases. The model is structured using the QUAIDS representation suggested by Banks, Blundell and Lewbel (1997), with demand estimated for four categories; Retail DSL, Retail PSTN, Wholesale DSL and Wholesale PSTN. For each category, prices are calculated as a volume-weighted average of the within-category products. With  $N=4$  products, there are  $3(N-1) + 0.5(N)(N-1) = 15$  parameters to estimate. Using the delta method to compute the covariance matrix, all estimated coefficients are found to be significant, with P-values of zero to three decimal places.

Table 4.1 Own and Cross Price Demand Elasticities

		$\Delta Q$			
		Retail DSL	Wholesale DSL	Retail PSTN	Wholesale PSTN
$\Delta P$	Retail DSL	-0.626	-0.272	0.697	0.201
	Wholesale DSL	-0.236	-0.134	0.288	0.082
	Retail PSTN	0.794	0.440	-0.696	-0.539
	Wholesale PSTN	-0.069	-0.075	0.177	-0.032

Source: Indecon analysis of Eircom Line Base Data

The results of the Quadratic Almost Ideal Demand System Modelling, shown above in Table 4.1 indicate a number of important relationships. As is to be expected, Retail DSL, Wholesale DSL, Retail PSTN and Wholesale PSTN services are found to have negative own price elasticities, with the effect of a 10% increase in the price of each service resulting in a reduction in volume ranging from -0.32% to -6.96%. Retail DSL and Retail PSTN own-price elasticities are found to be similar in the magnitude if their effects, with only 0.07 between the two values.

Cross Price Elasticities show the same directional effect between two sets of products, e.g., if a rise in the price of Product I causes a reduction in demand for Product J, then a corresponding rise in the price of Product J will cause a reduction in demand for Product I. Wholesale DSL & Retail DSL are found to be substitutable, have negative cross-price elasticities, while Retail PSTN & Wholesale DSL and Retail PSTN & Retail DSL are complementary, having positive cross-price elasticities. Of particular import are the effects of Retail DSL on Wholesale DSL and Wholesale DSL on Retail DSL. Both are negative, indicating that the products are complementary, and have the same order of magnitude; a 10% increase in the price of Wholesale DSL leads to a 2.36% reduction in the demand for Retail DSL, with a similar effect resulting from a 10% increase in the price of Retail DSL on the demand for Wholesale DSL (2.72%). A comparable effect is estimated to hold true for the effect of a price increase in Retail PSTN on Wholesale PSTN, although the magnitude of the effect (-0.539) is smaller than that found for DSL services.

The effect of VOIP may be driving the fact that retail DSL is a substitute for wholesale PSTN, but this is difficult to know for sure.



### 4.3 Summary

In this section, we have estimated the elasticities of demand for BB and PSTN fixed line service in Ireland using eircom monthly data on lines and prices. The key result is that the estimated coefficients are significant and that the own price elasticities of demand for DSL/BB and PSTN retail are very similar (-0.626 and -0.692 respectively).

The implication of this (and the conclusions of the Millar paper) is that based on available evidence the 50/50 rule is not likely to be too far from the optimal Ramsey-Boiteux rule.

We would note, however, that we believe that the 50/50 rule, and our empirical evidence, may need to be either updated over time or considered on a more forward looking basis. Eircom, noting the fall in PSTN lines, has legitimate concerns that PSTN lines, once inelastic, are becoming more elastic. The fact that eircom did not increase PSTN prices by the allowed amounts at the end of 2008 is clear evidence of this. We therefore would suggest that the appropriate starting point might be 50/50, but that with consideration of future dynamics, that a rule which allows for more flexibility might be achieved through further study and/or discussion between eircom and ComReg.

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## 5 Conclusions

In summary, ComReg is proposing that the share of fixed and common costs of the local loop in the rental price of line sharing in Ireland should be set to zero. (In other words, all the fixed and common costs would be recovered by voice line rental alone). ComReg's main arguments are based around the following:

1. The pure incremental cost approach is the 'best' approach, according to their consultants;
  - a. The incremental cost, given a subscriber has a voice (PSTN) line, is zero.
2. According to ComReg 08/106, the "most important and fundamental<sup>58</sup>" consideration is that eircom already is recovering its access network costs from voice (low frequency) line rental charges
  - a. Therefore eircom has no need to recover access network costs from LS rental charges.
3. Allocating zero of the fixed cost recovery to LS is not likely to negatively impact on the market, discriminate against certain technologies, discourage investment, competition, and technological development.

In summary, we believe that ComReg is either incorrect or has not sufficiently proven its case to reverse its previous determination that the structure of line share/PSTN rental prices should be set on a 50/50 basis. Our position is based on the following reasons:

1. The incremental cost approach has not been shown to be the best; a variant of Ramsey pricing is the best.
  - a. ComReg's own consultants have recognised that Ramsey pricing, or pricing that considers demand elasticities while neither under- nor over-recovering costs is the first best solution; however, they reject this as too difficult and not elsewhere implemented.

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<sup>58</sup> Page 8. "The most important and fundamental consideration is that the cost of a local loop on a bottom up long term incremental cost ("BU-LRIC") basis is already fully recovered through the price charged for narrowband access services ..."

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- b. A vast body of international research suggests Ramsey pricing, or variants thereof, is the best, and that this is a very common form of pricing in the international network utility setting in OECD countries.
  - c. We estimate elasticities of voice and broadband services in Ireland. The estimates suggest that while uncertainty exists of the 'exact value', there is little uncertainty that voice services are sufficiently elastic such that allocating all of fixed access costs to them would be inefficient.
  - d. ComReg's claimed basis of their judgment to choose incremental costing (IC) as the pricing basis for LS is not sound. The claimed split of ComReg's own consultants is that 10/18<sup>59</sup> selected countries use IC. The methodology is highly arbitrary and sensitive to the sample. Inclusion of more natural comparators<sup>60</sup> to Ireland would mean only 10/22 countries use IC, and only 3/22 use a zero allocation of fixed access costs to LS.
2. ComReg's current claim that tariff rebalancing implies a zero cost share for LS in the structure of charges is logically unfounded; the efficient level of cost recovery tells one nothing in particular about the efficient structure of costs.
    - a. The level of costs is not a relevant issue for this consultation *per se*; the structure of costs is. Eircom's overall level of cost recovery for its access network and the *most efficient structure of charges* between PSTN and LS are two wholly separate issues.
    - b. ComReg should, according to its duties and responsibilities, determine independently:
      - i. The efficient levels of cost recovery for services which are joint and common to fixed network assets
      - ii. The efficient structure of charges

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<sup>59</sup> The EU 15 plus the USA, Japan, and Australia.

<sup>60</sup> Including NZ, Canada, Switzerland, and Norway.

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- c. Eircom is not now, was not previously, and in the future is not seeking to over-recover its access network costs. Eircom proposes that if there is an issue of over-recovering with its access network charges, then they would be happy to agree an overall level of charges that minimises the possibility of over-recovery to an acceptable level.
  - d. Indecon/LE submit that correctly, Eircom's sole concern here is with the structure of charges/the split of fixed cost recovery between voice and LS. Eircom merely have a legitimate concern that voice line rental is rapidly becoming a more elastically demanded product and they need pricing flexibility to react to market dynamics and uncertainty. Eircom retail PSTN line rentals have fallen 26% from their peak (eircom data), all the while the number of Irish private dwellings has increased about 25% over a similar period (2004-2007 CSO data).
3. ComReg's current proposal that LS rental charge access network cost share of zero is contrary to ComReg's own objectives, as it is likely to negatively impact the market, discriminate against certain technologies, and risks 'choosing winners' (e.g., LS over Cable BB, Mobile BB, Satellite BB, FTTH, etc), negatively impacting particular operators and technology users, and is overall likely to be inefficient<sup>61</sup>,<sup>62</sup>.
    - a. The evidence produced by ComReg's own consultants ComReg 08/106a suggests low LS prices reduce investment.
    - b. A LS price that is artificially low will negatively impact Cable Broadband (BB), which is both the fastest growing part of the BB in Ireland and in many countries the most significant BB competitor.

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<sup>61</sup> One of ComReg's objectives under the Communications Regulation Act 2002 is to promote competition including ensuring that there is no distortion or restriction of competition.

<sup>62</sup> The inefficiency arising from the fact that demand elasticities have not been fully exploited in the setting of the share of fixed and common costs to be recovered between two joint products: voice and LS.

- c. Eircom proposes that a reasonable approximation to a Ramsey pricing policy could be reached within a reasonable time with ComReg. The approach would be based on agreement on a) the overall *level* of charges/cost recovery (akin to the Ramsey number) and b) the structure of charges (based on ratio of inverse elasticities).

## 6 Annex: Derivation of the QAIDS Model

This annex describes the derivation of the QAIDS, or quadratic aids model. The quadratic aids model is slightly more sophisticated than the linear AIDS model, but allows a less restrictive set of demands to be estimated as it provides a quadratic in the logarithms approximation to the demand system, as opposed to a linear in the logs system (both systems allow non-linearity in demand).

Let the expenditure share for product  $i$  be defined as

$$w_i = \frac{P_i q_i}{m}$$

where  $p_i$  is the price paid for product  $i$ ,  $q_i$  is the quantity of product  $i$  purchased, and  $m$  is the total expenditure on all products, such that

$$\sum_{i=1}^K w_i = 1$$

where  $K$  is the number of products in the demand system

Under the QUAIDS model, expenditure shares take the form

$$w_i = \alpha_i + \sum_{j=1}^K \gamma_{ij} \ln p_j + \beta_i \ln \left( \frac{m}{P(p)} \right) + \frac{\lambda_i}{b(p)} + \left( \ln \left[ \frac{m}{P(p)} \right] \right)^2$$

where  $p$  is a vector of all prices and  $b(p)$  is defined as

$$b(p) \equiv \prod p_i^{\beta_i}$$

and  $\ln P(p)$  is a price index defined as

$$\ln P(p) \equiv \alpha_0 + \sum_{i=1}^K \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^K \sum_{j=1}^K \gamma_{ij} \ln p_i \ln p_j$$

Given that  $\sum_{i=1}^K w_i = 1$ , it must be the case that

$$\sum_{i=1}^K \alpha_i = 1 \quad \sum_{i=1}^K \beta_i = 0 \quad \sum_{i=1}^K \lambda_i = 0 \quad \sum_{i=1}^K \gamma_{ij} = 0 \quad \forall j$$

And since demand functions are homogeneous of degree zero in  $(p, m)$

$$\sum_{j=1}^K \gamma_{ij} = 0 \quad \forall j$$

Slutsky symmetry implies that

$$\gamma_{ij} = \gamma_{ji}$$

As the matrix of error terms for the  $K$  equations is singular, one of the  $K$  demand equations is dropped from the system and the parameters of the final equation are recovered. The concentrated log-likelihood function for the  $(K - 1)$  equations in a sample of  $N$  observations is

$$\ln L = -\frac{N}{2} \ln |S| + \ln(2\pi) + \ln |S|^{-1}$$

where  $N$  is the number of observations and

$$S \equiv \frac{1}{N} \sum \hat{\varepsilon}_t^* \hat{\varepsilon}_t^{*'}$$

where  $t$  indexes time periods and  $\hat{\varepsilon}_t^* \equiv [v_{1,t} - \hat{w}_{1,t}, \dots, w_{K-1,t} - \hat{w}_{K-1,t}]'$

## A1.1 STATA Outputs

	Coef.	Std.	Err.	z	P>z [95%	Conf.
alpha						
1	-2.21812	0.19664	-11.28	0	-2.60353	-1.83271
2	-0.80839	0.171458	-4.71	0	-1.14444	-0.47234
3	7.973342	0.268125	29.74	0	7.447827	8.498857
4	-3.94684	0.363989	-10.84	0	-4.66024	-3.23343
beta						
1	0.034948	0.013294	2.63	0.009	0.008893	0.061003
2	-0.02208	0.010618	-2.08	0.038	-0.04289	-0.00127
3	-0.32939	0.01998	-16.49	0	-0.36855	-0.29023
4	0.316527	0.025049	12.64	0	0.267433	0.365622
gamma						
11	-0.96689	0.100067	-9.66	0	-1.16302	-0.77076
21	-0.37178	0.046315	-8.03	0	-0.46256	-0.28101
31	1.872809	0.153999	12.16	0	1.570977	2.17464
41	-0.53414	0.084232	-6.34	0	-0.69923	-0.36904

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	22	-0.17669	0.036728	-4.81	0	-0.24868	-0.1047
	32	0.735731	0.112289	6.55	0	0.515648	0.955814
	42	-0.18726	0.051712	-3.62	0	-0.28861	-0.0859
	33	-4.54171	0.301434	-15.07	0	-5.13251	-3.95091
	43	1.933173	0.238949	8.09	0	1.464841	2.401504
	44	-1.21178	0.192659	-6.29	0	-1.58938	-0.83417
lambda							
	1	0.009259	0.000664	13.95	0	0.007958	0.01056
	2	0.005732	0.000578	9.93	0	0.004601	0.006864
	3	-0.01236	0.001318	-9.38	0	-0.01494	-0.00977
	4	-0.00264	0.001314	-2.01	0.045	-0.00521	-5.9E-05